Tailoring Guide

for the

Secondary Mathematics Curriculum

Prepared by the Curriculum Development Council
Recommended for Use in Schools by the Education Department
Hong Kong
1996

TAILORING GUIDE FOR THE SECONDARY MATHEMATICS CURRICULUM

Introduction

The present Secondary Syllabus for Mathematics has been designed for the whole population of S1-S5 students, which covers a wide range of abilities, interests and needs. To assist teachers to tailor the mathematics curriculum to meet the needs of their individual groups of students, it is considered useful if a 'Tailored Part' of the Syllabus could be identified, which represents the essential part of the Syllabus that ALL students should strive to master. The rest of the Syllabus would constitute additional levels of learning teachers may prescribe for students according to their abilities, interests and needs.

Criteria for Identifying the Tailored Part of the Syllabus

The CDC Mathematics Subject Committee (Secondary) re-examined carefully the existing Secondary Mathematics Syllabus in order to identify topics for inclusion in the Tailored Part. Consideration has been given to the academic, social, vocational or personal relevancy of the topics to the students. The intention was to form a 'Tailored Part of the Syllabus' of learning which should

(a) be the minimum body of learning for every student,

(b) contain different components that constitute a coherent curriculum,

(c) emphasize on important knowledge, concepts and skills.

Tailoring Arrangements

Following the above criteria, attempt has been made to identify in the current Syllabus knowledge, concepts, terminology, processes and skills which are considered to be of basic nature or of particular importance. These should be supplemented by graded examples the teacher would give for illustration in class and exercises the teacher would assign to students for consolidation of the learning, including applications to daily life problems and problems in mathematics itself.

Enrichment topics related to the above or other unrelated topics which are more complex or advanced (but not essential) in nature will form additional levels of learning outside the Tailored Part. For ease of reference of teachers, these topics are enclosed in *dotted boxes* in the attached Tailoring Guide, which is reproduced from the existing Syllabus document, page for page. Notes and additional remarks (enclosed in *solid boxes*) are also given in the margin of the Guide to delimit the complexity of teaching of certain topics to the average or low-achieving students. Using the time allocation suggested in the original Syllabus as a quantifier of the workload involved, topics in the Tailored Part constitute some 70% of the total workload in the whole syllabus.

It should be emphasized that identifying the Tailored Part from the Syllabus is only the first step in the process of curriculum tailoring. With a clear knowledge as to which parts of the Syllabus are considered more essential, teachers should now feel more comfortable in making a decision to limit the scope of their teaching.

It is very important that students be provided with the opportunity to learn through their own experience and at a level appropriate to their own achievement. The success of curriculum tailoring will therefore hinge on teachers' ability to adopt teaching strategies and choose learning materials most appropriate to the characteristics of their students. Thus, for example, in teaching a particular mathematical concept or process, the degree of formalness or abstractness in the treatment will depend on the mathematical maturity of the students. The examples for illustration and exercises for consolidation will also depend on their personal experience or level of achievement.

It must also be stressed that the Tailored Part is *not* a rigid body of learning and teachers should not feel obliged either to teach the Tailored Part or to teach the whole Syllabus. Teachers should judge for themselves the suitability and relevance of topics outside the Tailored Part for their own students.

Re-allocation of Time Ratios

The time allocated to the teaching of the subject should remain the same, no matter whether the teacher is carrying out curriculum tailoring or not. As the Tailored Part is only a minimum but not a *rigid* body of learning for every student, it is considered *not* desirable to re-allocate time ratios to individual topics it contains. Teachers may, apart from teaching this part of the Syllabus, include some additional levels of learning in their teaching. They should therefore judiciously re-assign the time saved from not teaching certain additional levels of learning to the teaching of topics in the Tailored Part.

All comments and suggestions on the Guide may be sent to :

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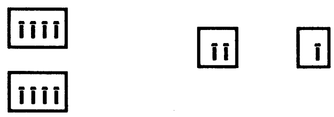
UNIT 1

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
1	**Numbers and counting** *Objectives:*	1.1 Revision on arithmetic.	4	The 4 rules, simple fractions, H.C.F. and L.C.M. should be revised. The meaning of prime numbers should also be brought into review. Number work should be restricted to positive numbers.
	(1) To revise the number work done at primary level. (2) To understand the relationship between numbers and numerals. (3) To see the advantages of Hindu-Arabic numerals and to accept them as part of our cultural heritage.	1.2 Primitive man's concept of numbers and the different systems of numerals used to represent counting numbers in the early civilizations.	1	Students may be interested in the rich variety of numeral systems that have existed. Some of these are still widely used today such as Chinese numerals while others are found only in history books. It is not essential for students to be able to convert a number in one system to another system. Teachers can set out a simple multiplication using Roman numerals just to illustrate how impractical this system is, when it comes to calculation. Stories of how primitive man understood the concept of numbers and how these numbers evolved from knots on a string to written symbols should be of interest to students. We know that counting was practised by man long before the invention of numbers.
	(4) To recognize some basic properties of natural numbers. (5) To work on binary numbers.	1.3 The advantages of the Hindu-Arabic numerals over other systems and the creation of zero as a place holder.	2	Students are expected to know the difference between digit and place value and they should be able to represent any denary number as a sum of units, multiples of 10 and multiples of 100, etc. (avoid using exponents unless students are already familiar with them). It can be pointed out that in some numeral systems, an empty space is used instead of the place holder "zero". Also at this stage it may be of interest to introduce some basic idea of binary numbers since these numbers are essential to modern computers. It can soon be seen that while a denary number employs 10 symbols, a binary number employs only two, 0 and 1. The point to emphasize is the idea that "rather than 2 in a column we carry 1 to the next column". To illustrate this point, teachers may use objects like marbles, toothpicks or matches and put them in groups of 8, 4, 2 and 1 like  which is represented by 1011.

UNIT 1

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
1				Why don't we group them like  and represent the number by 211? Teachers may like to illustrate the significance of binary numbers using mechanical devices such as punch cards, switches and flash light bulbs.
		1.4 Conversion of denary numbers into binary numbers and vice versa; addition and subtraction of binary numbers.	5	Conversion between the denary and binary numbers should be limited to simple numbers only and fractions should be excluded. Addition and subtraction of the binary numbers usually fascinate students who first encounter this system. It is desirable that they are given some practice in these two simple operations. The use of simple teaching aids and activity method to introduce this topic is recommended.

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2	**Formulae, open sentences and simple equations** *Objectives:* (1) To learn the use of letters to represent numbers. (2) To understand the difference between a mathematical sentence and an equation. (3) To construct and solve simple equations and apply them to practical problems.	2.1 The use of letters to represent numbers.	2	The use of letters to represent numbers arises quite naturally when formulae such as A = b×l and P = 2(b+l), where A stands for area, b breadth, l length and P perimeter of a rectangle, are considered. Teachers can point out that even in daily life, letters may be used to represent numbers, e.g. in a secret code. Your ticket is A1234; does the letter A mean anything? Teachers should stress that throughout the algebra course, letters represent numbers.
		2.2 Formulae and substitution.	3	Familiar formulae can be mentioned and it should soon be evident to students that in these simple formulae when all but one of the letters are replaced by numbers, the remaining letter (called the unknown) can be calculated. This process of replacing letters with numbers is called substitution.
		2.3 Open sentences.	3	A statement about numbers (or anything to do with mathematics) is a mathematical sentence. An open sentence is one which can be either true or false depending on how the blanks in the sentence are filled. Teachers may first introduce some simple sentences like ♦ is 4 years old, ( ) is round. Then open mathematical sentences can be considered. e.g. $\bullet+3=11$ $\frac{\blacksquare}{21}+5=7$
		2.4 Simple linear equations in one unknown, their construction and solution in practical problems.	4	It follows from the above that it is more convenient to replace the blanks by letters like x and y. Students can observe that different letters are used to replace different shapes of blanks and when there is an equal sign "=" appearing in an open sentence, it becomes an equation. They should soon discover that when an equation has two or more letters, then the values of these letters are usually indeterminate. For simple equation with one unknown, the standard

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2				form is ax+b=0, where a and b are constants. After students have sufficient practice in constructing and, subsequently, solving simple equations in connection with practical problems, teachers may then introduce the terms: "variable", "coefficient", "solution", "term", etc. in an equation. If only very simple numbers are used in exercises, then there is no need for the rules of transposing terms in an equation. This tends to be confusing, rather than illustrative, at this stage. Checking of answers should always be encouraged.
			12	

FORM I

UNIT 3

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
3	**Use of protractor and compasses and basic properties of angles and simple shapes** *Objectives:* (1) To approach geometry in an informal and practical way. (2) To appreciate the practical use of geometry. (3) To work on problems related to angles, congruence and similarity.	3.1 Use of the protractor and ruler to measure and construct angles in any position and use of the compasses to mark off length.	3	The use of the protractor may already have been taught in Primary School; nevertheless teachers are advised to teach it again and ensure that every student in the class can use the instrument without error. Angle is a fundamental idea and whether it has been approached as a rotation or fraction of a revolution its measurement helps the understanding of the concept. The measurement of an angle can best be taught using an overhead projector as the students' protractor can easily be shown on the screen. Failing this, a blackboard protractor is useful. Throughout this introductory unit in Geometry, it is advisable to leave terms like "line segment" and "angle" as undefined terms and to accept where appropriate whatever intuitive ideas the students may have about these terms.
		3.2 Acute and obtuse angles and the two scales of the protractor.	2	For some classes, teachers may find it helpful to instil the idea of acute and obtuse angles before using the protractor in order to help student choose which of the two scales to use on the instrument.
		3.3 Illustration of angle sum of a triangle.	1	By tearing off corners from a triangle and laying them on a straight line, students should be able to see that the angle sum of a triangle is 180°.
	N.B.: In 3.4, no attempt should be made to prove the congruence or similarity of triangles.	3.4 Congruence and similarity investigated through the construction of triangles.	7	From one set of instructions, each student should construct one triangle (and this may be orientated in any position). The class should then cut out their triangles so that the teacher can stack them into a triangular prism. What must we do to them before it is possible to stack them? What does this property of stacking illustrate? Similarity should be introduced through examples in the environment, and its properties should be limited to triangles only.
			13	

UNIT 4

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
4	**Percentages** *Objectives:* (1) To see percentage as a particular form of fraction and to change percentages into fractions and vice versa. (2) To apply percentages to practical problems.	4.1 Meaning of percentage.	1	Emphasis here should be on the meaning of percentage. Students should see the need for using percentage as a convenient means for comparing fractions. This can be done through consideration of some realistic problems such as discounts. Percentage is then understood as a particular fraction where the denominator is 100, hence the term "per cent" (i.e. per hundred). While there are many ways of comparing fractions, conversion to percentage is the most practical way and is especially acceptable to the layman in dealing with problems in commerce or technology.
		4.2 Practice in converting fractions to percentages and vice versa; decimalization of the fraction can be considered as an intermediary step.	2	Practice should involve simple numbers only. Graded exercises in manipulation can be given to test accuracy and mental drills can also be used as a diagnostic tool where mastery of the skill is in doubt.
		4.3 Percentages in everyday problems: interest rate, discount, profit and loss, etc.	7	As students are already quite familiar with these topics, they should try some harder and more practical problems to consolidate the work done at primary level. Problems should be simply worded so as not to obscure application of percentages.
			10	

UNIT 5

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5	**Simple areas and volumes** *Objectives:* (1) To find the areas of polygons. (2) To find the volumes of solids with uniform cross-sections.	5.1 Comparison of areas, measurement of area and unit area.	2	The idea of area is a fundamental concept that has been dealt with in Primary School. To develop this concept, students should compare the areas of figures with similar or different shapes. It soon becomes apparent that a standard unit is required for comparison. This unit area usually takes the form of a square, triangle, hexagon or any other shape that tessellates.
		5.2 Areas of simple polygons, including regular and irregular shapes; use of pinboard for irregular polygons.	4	A very convenient and inexpensive piece of apparatus, useful for this topic, is the pinboard (geoboard, pegboard). As a substitute for the pinboard, students can use squared paper. Areas of a variety of simple polygons can be investigated by drawing the polygons on squared paper. When a pinboard is used, it is a good idea to try to discover the formula $(\frac{1}{2}m-1)+n$ where m denotes the number of pins on the boundary of the polygon and n the number of pins inside the polygon. Teachers may guide and help with formulation, step by step, using many examples. No formal proof need be given.
		5.3 Unit volume.	2	Here again, students should realize the need for choosing a standard unit of volume. They should also realize that the unit volume should tessellate.
		5.4 Volumes of cuboids and solids with uniform cross-sections.	4	The volumes of cuboids, solids of uniform cross-sections and the same height but varying bases (equilateral triangle, square, regular hexagon, regular octagon) should be investigated. Some students may be interested in building frames of solids using plasticine and toothpicks. For high ability groups, it may be worthwhile to investigate these networks and as an activity students can attempt their design on paper. As a puzzle or a game, teachers may also mention Euler's formula: $V+F-E=2$.
			12	

UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6	**Approximation and measurement** *Objectives:* (1) To understand the meaning of measurement. (2) To practise in measuring.	6.1 Approximation; measurement.	3	Students should see that all measurements are approximations. The more appropriate measuring device we use, the greater is the degree of precision. Take, for example, a leaf whose traced outline encloses approximately 150 squares on a sheet of graph paper. This is not an exact number, as in counting the number of people in a classroom, because we have to think of parts of squares adding up to whole squares to arrive at our total. This type of counting is an approximation, just as any measurement is an approximation. Teachers can point out that, for practical use, such as measuring the length of this paper, a precision to the nearest millimetre is sufficient. Practical activities should be assigned whenever appropriate. The symbol "≈" should be used where an approximation is intended.
		6.2 Choice of appropriate unit for measurement.	1	It is desirable to mention that there are many units for measurement. But it should be pointed out that the appropriate size of unit should be chosen for certain measurement. For example we use m^2 for measuring the area of the school playground but cm^2 for the area of a desk.
			4	

UNIT 7

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
7	**Negative numbers and the extended number line** *Objectives:* (1) To understand and to accept intuitively the concept and uses of negative numbers. (2) To handle negative numbers in calculations.	7.1 Introduction of negative numbers as a means to solve some linear equations.	2	Students should see the necessity for introducing negative numbers. An intuitive interpretation of negative numbers representing deficit, temperature below zero, time before count-down, and so on can then be discussed.
		7.2 The complete number line and calculation with the help of the number line.	4	With the introduction of negative numbers, the number line, which is a graphical representation of numbers, can be extended at both ends. The calculation of -5-7 can be performed on the number line to give an answer of -12. Numbers are now divided into 2 groups, the positives and the negatives (0 is unique).
	N.B.: More illustrations with daily life examples could help understand the concept of negative numbers.	7.3 Simple idea of ordering.	3	This is a natural development from the number line. Students should soon discover that any number lying to the left of a certain number on the number line is smaller than any number lying to the right. The symbols ">" and "<" can be introduced and students can conclude that $-7<-5$ and $7>5$ Can students find any practical explanation for $-7<-5$?
			9	

UNIT 8

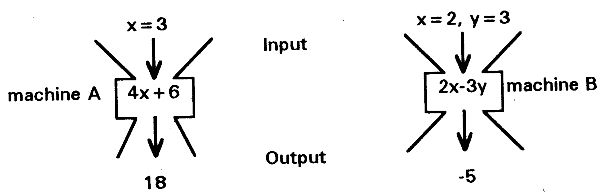
Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8	**Introduction to coordinates** *Objectives:* (1) To learn another type of geometry—coordinate geometry. (2) To understand the concept of an ordered pair of numbers. (3) To learn the calculation of distance and area in coordinate geometry.	8.1 Locating a point in a plane by means of an ordered pair in a coordinate system.	3	This involves two concepts: (a) reference lines leading to a grid system; (b) some sort of ordered pair to represent a point on the grid. These ideas can be introduced by asking the class to describe the position of a particular student in the classroom. The obvious answer gives the student's position referring to columns and rows. This can be refined and subsequently an idealized grid system formed. Examples of other grid systems, such as latitude and longitude, atlas notation for maps, games such as "battle ships" and chess can be used. By this time the idea of an ordered pair and a coordinate system should be emerging. At this stage the distinction between an ordered pair describing a square of the lattice and a point of intersection of two grid lines should be emphasized and discussed. Which system describes a position more accurately? Then why is the other system also used?
		8.2 Use of rectangular and polar coordinates.	4	A great deal of oral work using a graph board, or grid on a projector, should be done at speed until there are no errors in the ordering of the number pair. Intuitively the coordinates of points on the grid can be asked for and then points plotted from given coordinates. The order of the pair should be emphasized and the reason for the order discussed. The same sort of exercises should now be given as written work to be done on graph paper. At this stage students should be challenged and asked for other means of describing position. Guidance may have to be given in order that they discover the idea of polar coordinates. A polar grid on an overhead projector would be very useful. If not, rather than spending time on the board, students may proceed themselves to polar graph paper. The rectangular and polar grids should be compared. Again an overhead projector is very useful for this type of work. With the aid of transparencies and overlays, work on the formation of grid lines, points on a line, naming lines on the grid, intersection of lines, regions and intersection of regions, plotting points to form lines and curves can be done efficiently and quickly.

UNIT 8

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
8				When learning polar coordinates, as long as students can plot a point using polars, this is sufficient for the less able. Further practice in plotting polar coordinates to form spirals may be given. The more able students may be able to appreciate that the grid as well as the numbered pairs determine the shape of the curve, and that there are reasons for using one grid in preference to another.
		8.3 Calculation of distance and area.	3	This sub-unit refers to calculations from first principles upon the rectangular grid and enables teachers to use the negative number unit that has just been studied. At this stage students have not done Pythagoras' Theorem and cannot calculate the distance between any two points. They should be given only questions involving vertical and horizontal distances. Area should be calculated from first principles and only those shapes that can be broken down into rectangles and triangles should be considered.
			10	

UNIT 9

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
9	**Algebraic expressions** *Objectives:* (1) To translate a statement into an algebraic expression. (2) To understand the nature of an algebraic expression that later leads to the idea of a function. (3) To see the connection between algebra and geometry as illustrated by graph of linear equation.	9.1 Practice in translating word phrases into mathematical phrases.	3	The number of days in x weeks; the cost of y apples at $0.5 each—these soon are translated into 7x days and $$\frac{y}{2}$. These are called algebraic expressions in one variable. Their significance may be discussed and it can easily be established that as soon as the values of the variables are assigned, the values of these algebraic expressions can b e determined. To what do we compare these expressions?
		9.2 The making and the use of linear algebraic expressions.	2	Teachers can draw on the blackboard diagrams like



Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
		9.3 Linear equation in two unknowns and its graph.	4	Students can conclude that an algebraic expression can produce numbers just as an ice-cream machine produces ice-cream. However, to get the output, some materials need to be put in first. In the case of ice-cream, it might be milk and sugar but in the case of algebraic expression, the input is number. From here teachers can easily lead the class to graph the linear algebraic equation. At this stage, only one equation need to be considered. It is desirable, however, to emphasize the idea that there are infinitely many ordered pairs satisfying the given equation.
			9	

FORM I

UNIT 10

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
10	**Angle and line segment bisection** *Objectives:* (1) To acquire a preliminary idea of loci. (2) To use some standard constructions as further applications of congruent triangles. (3) To acquire a first approach to the idea of proof.	10.1 Angle bisection using compasses and ruler.	4	Students may be asked to draw different types of angles, acute and obtuse, which may then be bisected using compasses and ruler. That the two halves of the bisected angle are equal can be explained by noting there are two congruent triangles. The given (dimensions constructed equal) can be marked upon the triangles, which are then folded upon each other, or cut out, then superimposed to illustrate the validity of the explanation. Reflex angles should also be bisected. A miscellaneous exercise involving bisection of all types of angles may now be introduced and students should be encouraged to use their protractors to check that the bisection is accurate. The term "angle bisector" should be emphasized and every student should know it is a line that bisects an angle into two equal halves.
		10.2 Construction of 90°, 60°, 45° and 30° using compasses and ruler.	3	The 60° angle construction follows from the construction of an equilateral triangle, and when this angle is bisected, we get an angle of 30°. The 90° angle can then be explained as a bisection of 180° and the 45° angle as a bisection of 90°.
		10.3 Construction of the perpendicular bisector of a given line segment using compasses and ruler.	4	The emphasis here is on the idea of perpendicular bisector (abbreviated as $\perp$ bisector) being a line that bisects the given line segment at an angle of 90° (which can now be called a right angle and written as rt.$\angle$). This construction can be verified by paper folding or cutting out the congruent triangles and superimposing them. The logical steps needed to verify the congruence can be written out to introduce students to a formal proof. Above-average students may be given exercises to find the in-centre, escribed centres and circumcentre of a triangle.
			11	

FORM I

UNIT 11

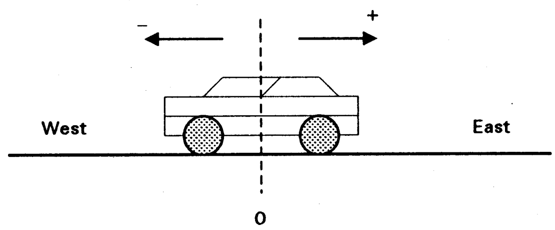
Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
11	**Angles and parallel lines** *Objectives:* To learn the properties of angles and parallel lines.	11.1 Adjacent angles on a straight line and angles at a point.	3	It must be impressed upon students that definitions cannot be proved. The difference between axiom and definition need not be discussed at this stage.
		11.2 Vertically opposite angles.	1	This is a simple proof which follows from adjacent angles. Students can use their protractors to check that they are equal.
		11.3 Parallel lines and transversal.	8	With the use of set squares, parallel lines can be constructed more rapidly and simply. Students should be able to see that the corresponding angles formed by a transversal cutting two lines are equal if the lines are parallel. If two parallel lines are cut by a transversal, the alternate angles formed are equal and the sum of interior angles formed is 180°. These may be derived from corresponding and vertically opposite angles.
	N.B.: Students should be given more guidelines in writing out the reasons.	11.4 The use of angles associated with parallel lines in calculations.	5	Throughout this unit, when numerical calculations are considered, simple numbers should be used, so that if students understand the reasoning and method, the question can be done quickly,. Initially, some questions may be answered orally. For harder questions, students are encouraged to set out their work in a clear, logical and economical way. Reasons should be given for each deductive step, but these should be kept brief, and a system of abbreviation agreed upon, e.g. alt. $\angle$s for alternate angles.
			17	

UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12	**More about negative numbers** *Objectives:* (1) To understand more about negative numbers intuitively. (2) To acquire a practical and intuitive meaning for the multiplication and division of negative numbers.	12.1 Operations with brackets: insertion and removal of brackets.	4	This is essentially an extension of Unit 7 and its chief aim is to enable students to deal efficiently with negative numbers in calculations. By now students should be quite capable of handling simplification of a long string of positive and negative numbers like −7+5−10+12. After that it is only a step further to introduce brackets into the expressions. Some teachers may prefer to distinguish clearly between the negative sign "−" in front of a negative number and the operation minus "−". Students may nevertheless be confused and find difficulty in applying even the simplest rules of signs in the insertion and removal of brackets. Hence it is recommended that only for high ability groups should teachers approach this topic in a rigorous manner. The average student readily accepts the type of argument that follows: $\because$ 12−(7−5) = 12−2 = 10 and 12−7+5 = 5+5 = 10 $\therefore$ 12− (7−5) = 12−7+5 For simplicity, there is no harm in categorising brackets into positive and negative. Their insertion and removal are then governed by rule, which if the students so desire, can be committed to memory.
		12.2 Intuitive approach to the multiplication and division of two negative numbers.	4	Teachers should provide one or two illustrative examples to give an explanation of the meaning of the product of two negative numbers. In these examples, generally two scales are necessary. For instance, if we take s km/h for the velocity of a car (going east as positive, going west as negative) and t hours (before as negative, after as positive) relative to a particular moment when the car passes O, a point of observation, then it is not difficult to see that when both s and t are negative, their product, i.e. the distance from O (east as positive, west as negative), is positive.

UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12	N.B.: Some other daily life examples might be more readily understood, e.g. rise and fall of temperature.			



As a further illustration to show the results of the multiplication of negative numbers, teachers can make out on the board a multiplication table as follows:

x	+3	+2	+1	0	−1	−2	−3
+3	+9	+6	+3	0			
+2	+6	+4	+2	0			
+1	+3	+2	+1	0			
0	0	0	0	0	0	0	0
−1				0			
−2				0			
−3				0			

Students should be guided to discover the pattern by completing the empty boxes, firstly those for the product of positive numbers and negative numbers and secondly, those for the product of two negative numbers. Similarly a division table can be constructed.

UNIT 12

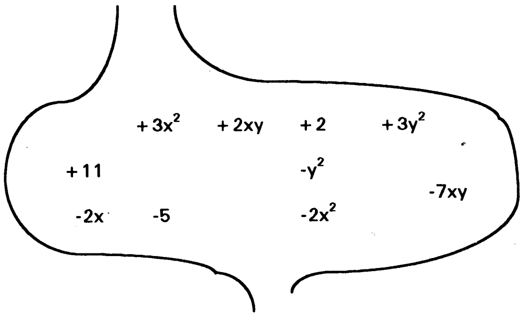
Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12		12.3 Justification of the above.	3	From the foregoing, a possible approach is: $\because$ 12−(0−5) = 12−0+5 $\therefore$ 12−(−5) = 12+5, and it follows that to subtract a negative number is in fact to add a positive number. Once this is accepted, it can be shown that $\because$ (−1)(5) + (−1)(−5) = (−1)x((5) + (−5)) (Distributive property) = (−1)x0 = 0 $\therefore$ (−1)(−5) = −(−1)(5) = 5 However, if students find this difficult to accept, teachers can always resort to defining the meaning of a product of two signed numbers.
			11	

UNIT 13

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
13	**Statistical data** *Objectives:* (1) To develop the ability for collecting data. (2) To understand the various ways of handling data. (3) To learn and to discuss the various methods of displaying data. (4) To understand the significance of statistical graphs and be able to draw conclusions from them.	13.1 Frequency and collection of data.	2	To begin with, students should be asked to collect data from their daily experience, e.g. the heights of the students in a class, the birth-months of the students in a class and so on. Special attention should be given to the organization and presentation of a large set of data. Difficulties in handling such data should be emphasized. This then leads to the idea of constructing a frequency distribution. Pre-planned work on an overhead projector is invaluable for this unit.
		13.2 Construction and interpretation of bar charts, pictograms and pie charts from given data.	3	Different kinds of data can be given and ways of handling them discussed. The use of tables, bar charts, pictograms and pie charts should be explained.
		13.3 Construction and interpretation of histograms.	5	Histograms may be regarded as a graphical representation of the frequency distribution. (These graphs may be constructed from activities such as practical surveying.) The meaning and the use of class boundary as well as the interpretation of histograms should be discussed in detail.
			10	

UNIT 14

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
14	**More about algebraic expressions** *Objectives:* To handle algebraic expressions more competently.	14.1 Simple idea of exponents.	3	Numerical examples should always precede statements of technique.
		14.2 Terms in an algebraic expression.	3	Using our analogy, an algebraic expression is made up of parts like a machine. What do we call these parts? Terms. Like and unlike terms can be viewed as like and unlike parts in a machine and every term has a sign in front of it. How do we collect the terms of the following expression?



Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
				The use of brackets in the simplification of algebraic expressions can thus be introduced. Students should have more practice in operation with brackets which include negative times negative or negative divided by negative and the simplification of algebraic expressions which may involve the "like" and "unlike" terms.
		14.3 Coefficients and constant terms.	2	It should be emphasized that the sign is attached to the coefficient and not the unknown.

UNIT 14

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
14		14.4 Addition, subtraction, multiplication of simple algebraic expressions.	5	Emphasis here is on technique and in the case of a product, long multiplication should be practised and in any case used as a check when the product can be written down immediately. For higher ability groups, divisions of one expression in one unknown by another simpler one in the same unknown can be considered. N.B.: Manipulation of algebraic expressions should be limited to those with a low degree and a small number of terms.
			13	
		Total:	153	

FORM II

UNIT 1

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
1	**Rate, ratio and proportion** *Objectives:* To develop the ability in the use of rate, ratio and proportion in problems connected with everyday life.	1.1 Meaning of rate, ratio and proportion.	3	Students are expected to understand clearly the meaning of rate, ratio and proportion through using everyday examples such as walking rate, reduction rate and the ratio of the number of boys to that of girls in a class. These examples should lead students to see their relationship.
		1.2 The notion of a two-term ratio a:b or $\frac{a}{b}$, where $b \neq 0$.	2	The notion of a two-term ratio a:b is introduced. This can be represented by the fraction $\frac{a}{b}$, where $b \neq 0$. Students should note that a ratio is unaltered if the two numbers (or quantities) of the ratio are both multiplied or divided by the same number. The notion of a two-term ratio may be extended to a three-term ratio or more, e.g. a:b:c = 1:2:3.
		1.3 Examples from science and mensuration including similar triangles. Problems on direct and simple inverse proportion. Graphs in two variables.	6	Students should be able to deal with rate, ratio and proportion in examples from science and mensuration, including similar triangles. Simple Practical problems on direct and simple inverse proportion should also be investigated. (N.B. Maps and scale plans are common examples of proportion.) Students may use graphs to see the relationship between two quantities.

FORM II

UNIT 2

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
2	**Angles of triangles and polygons** *Objectives:* (1) To investigate the basic properties of triangles. (2) To extend from simple to many-sided figures, particularly those occur in everyday life and those tessellate.	2.1 The angle sum of the interior angles of a triangle is 180°.	3	This has been introduced in 3.3 in Form I. By now students are expected to arrive at the same result using alternate and adjacent angles. Suffix notation is recommended as this helps to clarify the reasoning.
		2.2 The exterior angle of a triangle is equal to the sum of the two opposite interior angles.	3	This can be illustrated by cutting out a triangle, tearing off the two interior angles and fitting them over or into the exterior angle. The proof follows easily from the angle sum and adjacent angles. Throughout this unit, when numerical calculations are considered, simple numbers should be used so that if students understand the reasoning and method, the question can be done quickly. Initially, some questions may be answered orally (in which case each student should write down the answer).
		2.3 The use of small letters x, y, z, etc. to denote angles in a diagram.	4	Students should be encouraged to set out their work in a clear, logical and economical way. Rather than writing out the given, this should be marked in ink upon the diagram. Letters used for deduction should be marked in pencil on the diagram to differentiate them from those used for the given. Reasons should be given for all deductive steps but these should be kept brief and a system of abbreviations should be agreed upon, e.g. alt. ∠s for alternate angles.
		2.4 Polygons: interior and exterior angles of a polygon. Calculation of angles in a polygon by using the formulae: $\Sigma i = (2n-4)$ rt.∠s, $\Sigma e = 360°$.	7	This can be illustrated by dividing the polygon into triangles and, if necessary, cutting them out. Another method that can be used as soon as the sum of the exterior angles has been shown to be 4 right angles is to show that the sum of the exterior and interior angles is 2n right angles, hence the sum of the interior angles is (2n−4) right angles. With the exterior angles of a polygon, the most obvious illustration is cutting out the exterior angles and putting them together to form 4 right angles. (Some students may find it difficult to visualize a pencil turning through each exterior angle taken in turn makes a complete revolution. But of course this is quicker.) If the polygon is regular, and hence the exterior angles are all equal, we can easily obtain a formula for finding the size of each exterior angle.

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2				Also, if names are used for polygons, it is a good idea to quote the number of sides e.g. hexagon (6). When the formula work has been established, questions should be given in order to connect this topic with previous work. On its own it is merely and exercise in substitution. Construction of regular polygons can be done using any kind of equipment. In some cases it may be necessary to calculate angles and then use a protractor. Tessellation should be introduced informally as a form of tiling. Students should be shown how to decide whether a particular regular polygon tessellates. They should then discover which polygons tessellate and why they do so. These results should then be confirmed by constructing small pieces of tiling from the polygons, just large enough to show that the polygon concerned tessellates.
			17	

UNIT 3

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
3	**Approximation** *Objectives:* To understand the ideas of rounding off and significant figures.	3.1 Idea of rounding off.	2	A revision on the proper choice of units for measurement should precede the discussion on rounding off numbers. Students should know that numbers are rounded off for various reasons: for ease of reference or remembering or as a result of using inaccurate measuring instruments. The idea of rounding off should be illustrated, such as follows: If we use a ruler marked in tenths of centimetres, we can find the width of a sheet of paper, say 14.2 cm to the nearest tenth centimetre or 14 cm to the nearest centimetre. A sum of $2 578 spent on workers' welfare in a certain firm might well be rounded off to the nearest thousand dollars as $3 000 just for convenient reference. What is this sum of money when rounded off to the nearest (a) hundred dollars, (b) ten dollars? The number of children in a certain new estate is 3 864. To make it easy to remember the number might be rounded off to the nearest thousand as 4 000. What would it be to the nearest hundred?
		3.2 Significant figures.	2	Sometimes we round off numbers so that we can concentrate on the most important or significant digits. In the numbers 28.1 and 0.0281, which digit is the most significant? The most significant digits are always those whose place values are the greatest, that is, those on the left. The "2" is more significant than the "8" and the "8" is more significant than the "1" in the abov examples. When a number is given, it may be possible to tell the number of significant figures. For example, in a number like 0.0304 the first two zeros are not significant but the third is. Thus 0.0304 has three significant figures "3", "0" and "4". Also in a number like 17 50 (corrected to the nearest 10), it has four significant figures, namely "1", "7", "5" and "0", but the last "0" is not significant. To further the idea of significant figures, students should be encouraged to use them in practice and some examples should be considered. If the population of a new town is 287 850, it is good enough if we take it as 300 000. If you wanted more accuracy, you could use two significant figures and give the population as 290 000.

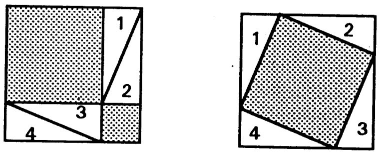
FORM II

UNIT 3

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
3				A flat is for sale at $645 700. A rich man might well think in hundred thousands of dollars. To him there is only one significant figure, the "6" in $600 000. A poorer man might worry about the hundreds of dollars. To him, there are four significant figures "6", "4", "5" and "7" in $645 700.
			4	

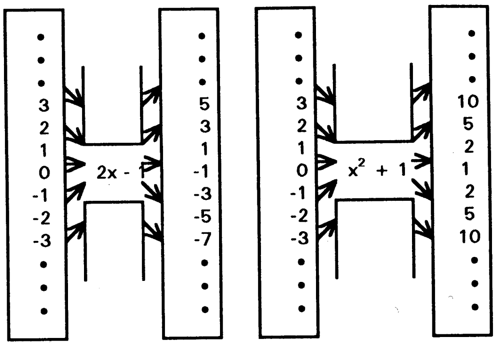
FORM II

UNIT 4

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
4	**Pythagoras' Theorem: use of square root tables** *Objectives:* (1) To apply the Pythagoras' Theorem in solving problems. (2) To see the importance of Pythagoras' Theorem in the study of coordinate geometry and other topics	4.1 Illustration of the veracity of Pythagoras' Theorem.	2	There are over three hundred proofs of Pythagoras' Theorem. However, the following illustration based upon equivalent areas obtained by the difference between a fixed square and four movable triangles is helpful. N.B.: Alternate proof of Pythagoras' Theorem is not necessary. 
	(3) To use the square root tables	4.2 Use of square root tables. N.B.: May use calculators	2	Use of the square root tables may be introduced through finding the length of an unknown side of a given right-angled triangle. A list of squared integers up to 20^2 is useful, as this gives the approximate value of any square root up to 400. This enables students not only to locate the decimal point but also to choose between the following type of alternatives when using the square root tables, e.g. $\sqrt{300} = 10\sqrt{3}$ and not $10\sqrt{30}$. To find the square root of a number, pairing off the digits and finding the square root of the first pair or first single digit confirms the first digit in the answer and shows which of the two alternatives to be selected from the tables.
		4.3 Application of Pythagoras' Theorem.	5	Simple calculations involving Pythagoras' Theorem should be introduced through applications such as carpentry. Application of the Theorem in simple problems in coordinate geometry and other topics should be discussed. N.B.: Limited to very simple problems.
			9	

UNIT 5

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
5	**Polynomials** *Objectives:* (1) To obtain a preliminary idea of a function. (2) To be familiar with techniques of handling polynomials.	5.1 From monomials to polynomials — viewed as number-producing machines.	3	This is an extension of unit 9 in Form I where students had experience in dealing with monomials. Students should understand how a polynomial is built up from a monomial (a link with Unit 14 in Form I where students dealt with collection of terms in an expression). To prepare students for the idea of functions in later work, teachers can emphasize one of the characteristics of polynomials, i.e. when the variables in a polynomial are assigned certain values, the value of the polynomial is determined. The correspondence is one to one or many to one but never one to many. Diagrams such as the following can be drawn to illustrate this point.



Students should know what is meant by the degree of a polynomial and be able to arrange the terms of a polynomial in ascending or descending order.

However, it is not recommended that terms such as "domain", "range", "image", "mapping", etc. be introduced, as these terms ar likely to confuse students at this stage.

FORM II

UNIT 5

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
5		5.2 Simple operations with polynomials.	4	Here the emphasis is on manipulative skills. Many worked examples should be shown before asking students to attempt the working. In any case the most complicated polynomials considered should not go beyond trinomial (perhaps, for high ability groups, polynomials of more than 3 terms can be attempted), not be with degree higher than three (Division of polynomials should be restricted to those in one variable). Long divisions like $(x^2+3x-1)\div(x-1)$ can be considered and students should get used to the idea of having a remainder when exact division is not possible.
	N.B.: Factorization should be limited to simple examples only	5.3 Factorization of polynomials by grouping terms.	6	Only the method of grouping terms needs to be considered here as preparation work for solving simple equations. While the general rule for grouping terms may be explained, it is not essential that students should commit the rule to memory. Students usually have their favourite means of spotting the factor or the terms that need to be grouped together, if they are given enough practice. As a check to see whether the factorization is correct, students can work out the product either in their mind or on rough paper.
		5.4 Simplification of algebraic fractions. N.B.: Denominators should be monomials	5	Finding the L.C.M. of two numbers should be reviewed (and perhaps the H.C.F. as well, though this is not required in this sub-unit). Students should be led to see the similarity between $\frac{2}{3}-\frac{3}{7}$ and $\frac{2b-1}{3a}-\frac{4b+2}{7a}$ the latter being an extension of the former. Once this is established, more complicated algebraic fractions can be attempted.

18

FORM II

UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6	**The sine, cosine and tangent ratios** *Objectives:* (1) To know the meaning and significance of some trigonometric ratios. (2) To use the tables for these ratios. (3) To apply these ratios in solving right-angled triangles. (4) To solve problems reducible to right-angled triangles.	6.1 Introducing the sine, cosine and tangent ratios for angles in $0° < \theta < 90°$.	4	The word "trigonometry" may be new to students. It is concerned with the measurement of angles. Teachers may consider the unit circle with its centre at the origin of a rectangular coordinate system. If P is a point on the circumference and OP makes an angle θ with the positive x-axis ($0° < \theta < 90°$), then for any specific value of θ, the x- and y-coordinates of P respectively give the cosine and sine ratios for the angle θ. A table with integral values of θ and the corresponding values of cosine and sine may be set up by students as an activity in class. The tangent ratio for the angle θ is given by the ratio of the y-coordinate to the x-coordinate of the point P. Students can then add the corresponding values of tangent to the tables they have set up.
		6.2 May use calculators Use of trigonometric tables.	3	For angles such as 35.5°, students may need trigonometric tables. Sufficient practice in looking up values from tables is essential. Although calculators may be used, students are advised to know how to use the trigonometric tables.
		6.3 Solution of practical problems reducible to right-angled triangles.	6	After some practice with the sine, cosine and tangent ratios, students should be guided to solve practical problems using right-angled triangles and trigonometric tables. Teachers may find examples from the measurement of inaccessible heights and distances such as the height of a tower or a tree, the distance between two cities and so forth. Emphasis here is on the discussion on ways of solving problems. Practical activities using simple surveying instruments should be conducted only when students are capable of mastering the trigonometric skills of problem-solving techniques.
			13	

FORM II

UNIT 7

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
7	**Trigonometric relations** *Objectives:* (1) To learn simple trigonometric identities. (2) To learn the trigonometric ratios of special angles.	7.1 Introducing the relations $\sin(90°-\theta) = \cos\theta$ $\cos(90°-\theta) = \sin\theta$ $\tan(90°-\theta) = \frac{1}{\tan\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta = 1$ students should know the relations $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$, but emphasis should not be put on their manipulation.	4	Teachers should prove the identities $\sin(90°-\theta) = \cos\theta$ $\cos(90°-\theta) = \sin\theta$ and $\tan(90°-\theta) = \frac{1}{\tan\theta}$ Exercises involving these identities should be given. Students may verify $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$ by measurement, calculators or trigonometric tables. This can be treated as an exercise for students. Teachers should then prove them using trigonometric ratios and the Pythagoras' Theorem. It should also be noted that with these two identities if any one trigonometric ratio of an angle is given, it is possible to calculate the value of any other ratio of that angle without using tables.
		7.2 Trigonometric ratios of special angles: 30°, 45°, 60°.	2	It is useful to introduce the trigonometric ratios of a few special angles. By using the Pythagoras' Theorem together with a right-angled isosceles triangle and an equilateral triangle, students should be able to obtain the trigonometric ratios of 30°, 45°, 60° and use them in radical form. Abler students may be shown a treatment of the trigonometric ratios for 0° and 90°.
			6	

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8	**Use of formulae** *Objectives:* (1) To appreciate the power of algebraic skills in comparison with arithmetic skills. (2) To practise the handling of literal relations.	8.1 Change of subject (radicals may should not be used).	5	A student who can transform formulae with three or four variables to obtain any assigned variable has a competent knowledge of the use of formulae, of simplifying expressions, of simple factorization method and of the solution of simple literal equations. Work in this unit should therefore be linked with these topics. Successful teaching of this unit often lies in the careful grading of examples so that each technique is mastered before the next is presented. The first transformations should be those of formulae containing two variables and these formulae should either be thoroughly familiar to or easily appreciated by students. An easy example is $S = 2N - 4$ (angle sum of a polygon); a harder example is $C = \frac{5}{9}(F - 32)$ $A = \frac{1}{2}(a + b)h$. One or two numerical illustrations may be worked on before the general transformation is attempted. From the outset it must be emphasized that the variable required must appear isolated on one side of the final statement. Students should not be discouraged from mastering the basic techniques of formulae transformation by being exposed to formulae which are too difficult for them to manipulate.
		8.2 Application of formulae: the method of substitution.	4	At this stage, students may have come across some formulae in their physics and chemistry mathematics lessons. Teachers are advised to look into students science mathematics course books for meaningful formulae both as examples or as exercises.
			9	

UNIT 9

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
9	**More about coordinates** *Objectives:* To learn the distance and slope formulae, and to understand further the idea of geometry as an algebraic structure.	9.1 Distance.	3	This unit follows directly from 8.2 in Form I and illustrates one of the many applications of Pythagoras' Theorem. Initially points should be chosen so that the horizontal and vertical distances between them form the sides of a Pythagorean triangle such as the rope stretchers' triangle (3, 4, 5). When students are sure of the principle involved, any two points can be taken and this will give practice in using the square root table. At this stage the generalized points (x_1, y_1) and (x_2, y_2) can be introduced and the formula obtained. With some classes, students may be able to do this themselves as an exercise.
	N.B.: This unit should be deferred until Form 3 and to be taught together with unit 5, p.68	9.2 Gradient.	3	This idea of slope, i.e. gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ for the line L passing through (x_1, y_1) and (x_2, y_2) should be discussed, noting that the gradient is independent of whichever point is taken first. Students should then consider both positive and negative gradients but compare only the positive gradient with $\tan\theta$ $(0° < \theta < 90°)$ where θ is the angle that the line L makes with the x-axis. The negative gradient will be compared later when the general angle is developed. Students should discover that parallel lines have the same slope. For abler students, the two cases when lines are parallel to the axes may be discussed. They may also discover from examples that the product of slopes of perpendicular lines is −1. The proof is not required at this stage.
			6	

UNIT 10

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
10	**Circle, rectangular block, prism and cylinder** *Objectives:* (1) To investigate ways of finding the circumference and area of a circle. (2) To find the length of an arc and area of a sector. (3) To solve practical problems on surface areas and volumes of solids.	10.1 Circumference of a circle. The approximate value of π. Length of an arc.	3	This is essentially a revision topic. Students should be encouraged to measure in metric units the circumferences and diameters of many circular objects such as tins, bottles and anything else with a circular cross-section. They can also draw a set of circles with the radius doubled each time. Teachers should guide students to discover whether a relation exists between the lengths of the diameter and the circumference. Statistical charts could be of some use to illustrate this relation. Students are expected to find the approximate value of the ratio. It may be of interest to introduce a brief history of the calculation of π. Length of an arc can be found as a fraction of the circumference using the ratio of the measures of the angles at the centre.
		10.2 Area of a circle and area of a sector.	3	Students may investigate the area of a circle by dissecting a circle into an even number of very small sectors in order to form a figure that approximates a parallelogram with base πr and height r and hence of area πr^2. The formula for the area of a circle can therefore be deduced. In calculating the area of a sector, students can be shown pie charts and led to calculate the areas of a quarter, half, three quarters and any sector of a circle using the ratio of the measures of the angles at the centre and hence the fraction of the area of the circle.
		10.3 Surface areas and volumes of rectangular block, prism and cylinder.	5	Volumes of solids with uniform cross-sections have been dealt with in Form I. To provide further practice, teachers should arouse students' interest by investigating various daily examples such as the volume of water in a swimming pool, water flow in a cylindrical pipe, increase in depth of liquid in a vessel when a solid is immersed.

UNIT 10

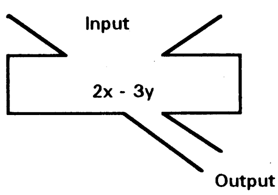
Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
10				Students should have no difficulty in calculating surface areas as it is a process of adding up area of triangles, rectangles, squares or circles. However, for high ability groups, it is challenging to investigate the relations between lengths, areas and volumes of similar objects. Students should investigate the relations between lengths and areas of similar objects.
			11	

UNIT 11

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
11	**Using percentages** *Objectives:* To use percentages to solve everyday problems.	11.1 Simple interest using direct proportion and the simple interest formula. Inverse problems on simple interest.	3	To study this topic, students should understand terms such as "Principal", "Rate", "Time" (in years), "Interest" and "Amount". It see that a table showing corresponding values of the above-mentioned items will best illustrate the fact that the simple interest is proportional to time. After some practice, students should discover the simple interest formula and make use of the formula in inverse problems.
		11.2 Compound interest as repeated simple interest.	3	Students should be led to discover the difference between simple and compound interest. They are expected to calculate compound interest through the repeated simple interest method. Teachers may find it easier to indicate step by step the method for computing compound interest in a tabular form. In some cases, it is advisable to obtain a rough estimate by calculating the simple interest for comparison.
		11.3 Knowledge of fixed deposit account.	2	In introducing compound interest, teachers may have already mentioned fixed deposit accounts in the bank. Interest is usually paid at fixed intervals: yearly, half-yearly or quarterly. For high ability groups, it may be possible to find out how interest is calculated for, say, ten days exceeding a fixed period of three months. For example, bank interest for those days exceeding a fixed period may be calculated on the seven-day-call interest rate.
		11.4 Growth and depreciation. N.B.: 11.4 should be deferred to F.3 & to be taught together with unit 1 on p.62.	4	As students are conversant with the calculation techniques involved, problems on growth and depreciation may now be investigated.
			12	

UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12	**Simultaneous linear equations in two unknowns** *Objectives:* (1) To practise the algebraic techniques acquired from Unit 9 in Form I and Unit 5 in Form II. (2) To learn how to solve simultaneous linear equations algebraically and graphically.	12.1 Simple algebraic methods: substitution and elimination.	6	This is an extension of Unit 9 in Form I and Unit 5 in Form II. Teachers are advised to refer to these two units before proceeding to this one. As an introduction, a simple linear polynomial in two unknowns can be written down and represented by a machine, thus:



What is the output for an input of $x=2$ and $y=1$? (The output is 1.)

Several questions of this type can be asked. It will soon be established that once variables x and y are assigned some definite values, the output value can be readily computed.

Can this process be reversed? That is, knowing the output, can we calculate the input? (This is possible with polynomials in one unknown.)

It soon becomes evident that with the above example, even though we know the output is -5, it is still not possible to conclude what values have been assigned to x and y. This enables students to see that another machine is needed. If there is another machine: $4x+3y$ at hand, and assuming the same set of input produces the output 17, then we have

$2x-3y=-5$ and $4x+3y=17$

both being true sentences simultaneously. Can the students then guess what the input should be?

This leads to the algebraic technique of solving simple simultaneous linear equations.

FORM II

UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12				The method of substitution provides an easy way for solving simultaneous linear equations. To make the work less tedious, it is advisable to choose the equation whose coefficient of x or y has the smallest absolute value. Equations that are inconsistent or that have no unique solutions should be avoided at this stage. The method of elimination serves as a useful way for solving simultaneous linear equations. In deciding whether to eliminate x or y, we should choose the one whose coefficients have a smaller L.C.M.
		12.2 Graphical method.	5	To further the idea, graphical method may be used. Students are already quite familiar with coordinates and the idea of the 1–1 correspondence between a point on the coordinate plane and the ordered pair (x,y). Now the simple technique of plotting a graph when an equation is given may be introduced. At this stage, students need only construct a table and plot the graph point by point. No lengthy discussion is necessary. When this is applied to simple simultaneous linear equations, it becomes evident to the students that the point of intersection gives the solution sought. As a further development, teachers may discuss with students cases where the straight lines are parallel or identical.
			11	

FORM II

UNIT 13

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
13	**Equations and identities** *Objectives:* (1) To distinguish between equations and identities. (2) To acquire the technique of expanding a binomial.	13.1 Meaning of an identity and the making of simple identities.	4	As a preliminary step, students should understand what is meant by the statement "The equation is satisfied by a certain value of x". In other words, the L.H.S. and the R.H.S. of the equation are numerically equal when a particular value is substituted for x. Students can then practise checking the roots they obtained for some equations like $\frac{x-1}{3}+4=9-\frac{2}{5}(3x-2)$. However, they should be prevented from proceeding as follows: $\frac{4-1}{3}+4=9-\frac{2(12-2)}{5}$ $\because 1+4=9-4$ $\because 5=5$ They should be made to realize that as a form of statement this is illogical since we are only checking the answer x=4 and we do not know it is true until the last line. They soon realize that identities are equations which are satisfied by all values of the unknown. Simple identities can then be considered and students should be able to construct some identities of their own, such as $(x-1)(x-2)=x^2-3x+2$, etc.
		13.2 The difference of two squares.	3	Following on from this, students will soon discover the identity $x^2-1\equiv(x+1)(x-1)$ and will accept readily the generalized result of $x^2-y^2\equiv(x+y)(x-y)$.
		13.3 Expansion of $(x\pm y)^2$.	3	This expansion is of great use for later work; hence more practice should be given to ensure that students master the technique.

UNIT 13

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
13				Expansion of a binomial to the power of 3 or above is rarely required. Nevertheless, it enables students to see the power of generalization. Besides, a discussion on Pascal's triangle is often stimulating and worth-while. Therefore, if time permits, teachers should at least mention the expansion of a binomial to some higher degree.
			10	

UNIT 14

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
14	**Frequency distribution and its graphical representations** *Objectives:* (1) To learn frequency distributions and cumulative frequency distributions. (2) To construct and to interpret the various graphical representations of the above distributions.	14.1 Frequency distribution, histograms, frequency polygons and curves.	6	Students should be asked to collect data from their daily experience. Histograms, frequency polygons and curves may be regarded as graphical representations of the frequency distribution.
		14.2 Cumulative frequency polygons and curves.	5	Similar approaches are then applied to cumulative frequency distributions and cumulative frequency polygons and curves. It is felt that the significance of the difference between grouped and ungrouped data should be stressed especially where there is a need to group the data in a frequency distribution.
		14.3 Interpretation of the above graphs.	4	Each statistical graph has its own characteristics. Teachers should prepare different types of graphs beforehand for demonstration and use overhead projectors wherever possible. Questions should be asked about the possible conclusions that can be drawn from a graph. Special attention should be paid to the suitability of a graph in providing information and conclusions of a certain type. Emphasis should be laid on interpretation and also the use of graphs for prediction. Further applications of those graphs will be discussed later.
			15	
		Total:	152	

FORM III

UNIT 1

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
1	**More about percentages** *Objectives:* (1) To learn the ideas of such things as direct taxation and percentage changes. (2) To get more practice in the use of percentages. N.B.: 1. Information on calculation of rates and direct taxes should be updated. 2. Unit 11.4 (p.56) of Form 2 shall be taught here.	1.1 Use of percentages in solving problems such as the calculation of rates and direct taxes.	3	This is an extension of Unit 11 in Form II. As students are conversant with the calculation techniques involved, problems on direct taxes such as profits tax, salaries tax, property tax and interest tax should now be investigated. The purpose of taxation should be explained clearly before giving practical problems based upon each type of tax. It should also be pointed out that taxes are charged at different rates, which may be changed from time to time. A copy of the table showing tax on net chargeable income from a demand note will be very helpful. Working through examples such as salaries tax for a married man with two children on a combined annual income of say $90 000 will provide students a lot of practice in the use of percentages.
		1.2 Use of percentages in solving problems such as the calculation of errors, percentage increase and decrease.	5	Percentage increase and decrease are introduced to quantitative changes. Problems may include percentage change in a quantity due to (a) successive changes, (b) changes in component quantities. For examples: (a) In a quotation for a car insurance, percentages of successive changes in premium are given: no claim discount: 60% surcharge of the premium as contribution to the central fund of the Motor Insurers' Bureau of Hong Kong: 1% If the basic premium is $1 000, what will then be the net premium? What percentage of the basic premium is the net premium? (b) Suppose the cost of a desk is calculated as follows: wood—$200, paint and sundries—$100, wages—$200. If the cost of wood is increased by 20% and the wages are increased by 10%, what is the percentage increase of the cost of a desk?

FORM III

UNIT 1

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
1				In addition to the above topics, students can study percentage error in relation is accuracy of measurement. The two terms "absolute error" and "relative error" need explanation. It should also be pointed out that the precision of a measurement is determined by the absolute error, the accuracy by the relative error.
			8	

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2	**Laws of indices** *Objectives:* (1) To learn the laws of indices. (2) To use the laws of indices in numerical exercises.	2.1 Indices and properties of indices.	3	Simple ideas of exponents are introduced in Unit 14 in Form I. It remains to provide at this stage a proof of the laws of indices where the indices are positive. It should be stressed here that students are expected to be convinced of the validity of the laws for negative and fractional indices. It is hoped that with this foundation students can be led to understand better the basic idea of common logarithms.
		2.2 Simple calculation involving rational indices.	3	The emphasis of this sub-unit is to reinforce the laws of rational (integral and fractional) indices by giving students some numerical examples. Graded numerical exercises can then be assigned for practice.
			6	

UNIT 3

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
3	**Common logarithms** *Objectives:* (1) To relate powers of 10 to common logarithms. (2) To practise the use of common logarithms.	3.1 Powers of 10 leading to common logarithms.	4	Students should have had plenty of numerical exercises in Unit 2. By now they should be relating the index of 10 to the common logarithm of a number. For example, $2 \approx 10^{0.30}$, the logarithm of 2 is approximately 0.30. This can be illustrated by referring to the graph of 10^x where x is any positive rational number. Logarithm tables can be used to explain how the logarithm of a number is obtained. Sufficient exercises should be given before proceeding to the next sub-unit.
		3.2 Practice in using common logarithms.	5	Students are expected to compute with common logarithm tables. Very often it is advisable to convert numbers to the standard scientific notation so as to avoid using negative characteristics. For example, to evaluate $\frac{27.2 \times 0.000\ 256}{0.001\ 23}$, students may change it to $\frac{2.27 \times 10 \times 2.56 \times 10^{-4}}{1.23 \times 10^{-3}}$ before using tables.
			9	

UNIT 4

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
4	**More about congruence, similarity and parallels** *Objective:* To understand the idea of deductive reasoning in geometry and to apply it to numerical problems.	General approach.		These topics have been dealt with separately and this unit is intended to knit them together in exercises whose object is to introduce a formal, yet less rigorous, type of proof. The idea here is to make this as easy for students as possible. Sketch and outline proofs may help students at this stage in order to show exactly what is required in a logical sequence of thought. In order to clarify and make more lucid the thought processes involved, teachers are advised to introduce a system of notations, abbreviations and a generalized layout and presentation of work. This makes for easier marking and enables teachers to follow students' proof more easily. In fact, it should enable students to follow their own proofs more easily, which is the point of the exercise. It is suggested that diagrams should be drawn in pencil, any constructions with dotted line. The given, where possible, marked in ink. Some marks should be placed on the diagram to show each piece of the given. The student then knows when thinking through his proof that he may consider each and every mark. It is recommended to use small letters x, y, z, etc. to denote angles in a diagram. In order to differentiate between the position of equal angles so that they can be easily referred to in the proof, a number suffix can be added to the base of the letter. Reasons should be given for each step but an agreed system of abbreviations should be used so as not to make the work laborious.
		4.1 Congruence.	9	The conditions for congruent triangles are to be reviewed. Application of these conditions on proving problems concerning congruence should be involved.

UNIT 4

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
4		4.2 Similarity.	5	After learning properties of congruence, students would like to learn the properties of similar figures. Projection of a plane figure on a parallel plane may give a very clear idea on similar figures and useful in demonstration. Then, the conditions for similar triangles may be investigated. Exercise on similar triangles should be given. Riders involving the ratio of areas of similar figures may also be assigned and discussed.
		4.3 Parallels.	5	By now students are quite familiar with parallels and conditions of congruence. Teachers may then give a definition of parallelogram and deduce its properties.
		4.4 Mid-point theorem and intercept theorem	6	The mid-point theorem is undoubtedly a very useful theorem in plane geometry. Teachers should derive the theorem from previous theorems and properties. Reasons should be given for each step. The intercept theorem concerning three or more parallel lines should be stated and proved. The special case of the theorem within a triangle should also be discussed. Exercises relating to mid-point theorem and intercept theorem should be given. It is stressed that students should have reason for making every statement or calculation.

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5	**Coordinate geometry of straight lines** *Objectives:* (1) To learn that an equation of the first degree represents a straight line. (2) To learn to write the equation of a straight line in standard forms. (3) To reinforce the ideas of slope and intercept. (4) To learn how to deduce a linear law from empirical data. N.B.: Unit 9 (P53) of F.2 to be taught here	5.1 Section formula (internal division): $x = \frac{sx_1 + rx_2}{r+s}$ $y = \frac{sy_1 + ry_2}{r+s}$ Coordinates of mid-point	3	Given a ratio r:s where r and s are positive integers students should work from similar triangles to obtain the point of division of a given line segment. After practice, the section formula can be derived. Then exercises relating to this formula may be given. At this stage, r and s should be confined to positive rational numbers. The mid-point formula $x = \frac{1}{2}(x_1+x_2)$, $y = \frac{1}{2}(y_1+y_2)$ may be regarded as a particular case of the section formula. Applications such as finding the centroid of a given triangle may be mentioned. To find the coordinates of the mid-point between two given points.
		5.2 Different standard forms of straight lines. Equation of a straight line	9	Whatever approach is used, whether it be through loci or sets, the underlying principle that should be emphasized is that every point on the line obeys a certain condition. This idea can be introduced by considering lines parallel to the x- and y-axes. Firstly, it should be emphasized that all points on the x-axis have a y-coordinate whose value is zero and that $y=0$ (where x is any value) is the condition that describes this and only this line and hence is called the equation of the line. This idea can be extended to lines parallel to the x-axis, such as $y=2$, $y=3$ and $y=1\frac{1}{2}$. The same sort of thing can be repeated using lines parallel to the y-axis.
		$ax+by+c=0$		Consider a line in the first quadrant that passes through the origin, e.g. $\frac{y}{x}=\frac{3}{2}$. It may be shown that, whatever point we take on the line, $\frac{y}{x}$ is always the ratio $\frac{3}{2}$, and that a point above the line would give a greater ratio, whilst a point below a smaller ratio. What happens when the line is "extended back" through the origin into the third quadrant? As soon as students realize that the same ratio applies and that $\frac{y}{x}$ still

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5				equals $\frac{3}{2}$, other examples should be taken including the case of a line, through the origin, but in the second and fourth quadrants. When students are sure that a line through the origin is either of the form $3x=2y$ or $3x=-2y$ then the generalized form $ax+by=0$ can be introduced. Can we generalize this further? Consider the two graphs $2x+3y=0$ and $2x+3y+7=0$. Students should plot these on the same piece of graph paper. All types of examples of pairs like this should be plotted. It should not take students long to discover that $ax+by+c=0$ is a line parallel to $ax+by=0$. What intercept does it make on the y-axis?
		$y=mx+c$		The students should by now realize that any line through the origin is $y=mx$ and that the y-coordinate of a point on the line measures mx. It can again be emphasized that a point whose y-coordinate is greater than mx is above the line, and a point whose y-coordinate measures less than mx lies below to line. In particular, points with y-coordinates that measure $mx+c$ lie at a vertical distance c above (or below) the line $y=mx$ and form a line parallel to it. This is really another way of looking at the underlying principle: the ordered number pair or coordinates of any point on the line are connected by a law. And points not on the line do not obey that law. The idea of points on a line satisfying the equation of the line can now be introduced. Students should be shown the equivalence of $ax+by=0$ and $y=mx$ and that m is the slope of the line. From the above work it should be also clear that c is the intercept cut off on the y-axis. Rapid practice examples, both oral and written, to determine slope and intercept should now be given.
		$\frac{x}{a}+\frac{y}{b}=1$		As this is an equation of the first degree it can be put into the form $y=mx+c$ which represents a straight line. Where does it cut the axes? Put $x=0$ and $y=0$ to find the intercepts. The points (a,0) and (0,b) are on the axes and determine the line. Rapid practice in determining the intercepts, given the line, and vice versa, should now be given.

UNIT 5

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5	N.B.: Students should know that the equation of a straight line is of the first degree, i.e. $ax+by+c=0$. Knowledge of equations in different form is not required. However, given two points, or one point and the slope, students should be able to find the equation of the straight line. On the other hand, given the equation of a straight line, students should be able to find its slope and intercepts.	$y-y_1=m(x-x_1)$		It may be shown that the slope of this line (equation of the first degree) is $m=\frac{y-y_1}{x-x_1}$, where (x_1,y_1) is a fixed point on the line and (x,y) is variable point. This form can also be compared with the form $y=mx+c$. Practice should be given in writing down the equation given the slope and the fixed point.
		$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$		If (x_2,y_2) is another fixed point (two points determine the line) then the slope m in the above form is $\frac{y-y_1}{x-x_1}$ which gives the required equation. Practice in writing down the equation of the line should now be given. Mixed sets of examples should now be given to include all the various forms. Intercepts and slopes should be written down given the equation and vice versa.
		5.3 Application: determination of laws.	4	When pairs of corresponding values of two quantities which obey (or are thought to obey) a linear law $y=mx+c$ are given, perhaps as experimental results, it is possible to test the conjecture by plotting these pairs of values. If the plotted points lie approximately on a straight line, then the students are justified in concluding that the law holds. The graph of the equation is taken to be the straight line drawn as evenly as possible between the plotted points. Discrepancies between the line and some of the points may be considered to be due to experimental errors. The constant m, that is the gradient of the line, can be found from any two points on the line, which may not necessarily be points given by the experimental results. The value of c can be found by reading the intercept on the y-axis. Alternatively, the constants m and c may be found by substituting the values of x and y at any two points on the line and solving the resulting simultaneous equations for m and c.

UNIT 5

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5				By suitable transformations, the same principle may be applied to test some non-linear laws such as $y=mx^2+c$; $y=m\sqrt{x}+c$; $y=\frac{m}{x}+c$, etc.

UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6	**Mensuration** *Objectives:* (1) To discover the relationship between the volume of a pyramid and that of a cube. (2) To learn the relationship between the volume of a cone and that of a pyramid. (3) To gain more practice in calculating volumes and surface areas of pyramids, cones and spheres. (4) To discover the relationship among volumes of similar solids.	6.1 Volumes and surface areas of pyramids.	4	The volume of a pyramid can be illustrated by showing how the cube can be divided into six congruent square-based pyramids. Teachers may induce students using a skeleton cube to discover this relationship. Students should then be led to realize that the surface area of a pyramid formed in this way is really a sum of areas of four congruent triangles and the square base. As an activity, students may try to make skeleton pyramids using plasticine and toothpicks or build up pyramids with paper and scissors. From this the volume of any pyramid can be deduced. Graded exercises should be given for practice.
		6.2 Volumes and surface areas of right circular cones.	4	A circle is the limit of a sequence of regular polygons. Similarly a cone may be considered as the limit of a sequence of pyramids. Students may be led to think of a cone as a pyramid with a circular base. This can be illustrated by a series of sketches or a set of polysterene models. A right circular cone can be formed by rotating a right-angled triangle about one of its shorter sides. Teachers should also encourage students to make cones with paper and verify the volume formula by filing the cones with sand. The area of its curved surface can be shown equal to the area of a sector of a circle. This can be illustrated by cutting along the slant height of a cone and unfolding it.
		6.3 Volumes and surface areas of spheres with formulae given.	3	This is essentially an extension of the above sub-units. With the formulae given, students can solve many problems such as calculating the volume and surface area of a football or basketball. Students may verify the volume formula of a sphere by water displacement or by filing in a hollow sphere with sand.

UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6		6.4 Ratio of volumes of similar solids.	4	By this time, students should be quite familiar with the idea of similar plane figures. Teachers may then extend the idea intuitively to similar solids. At this stage, it would be desirable to restrict only to similar regular solids such as cuboids, spheres, and cones etc. The relation between the ratio of the volumes of similar solids and the ratio of their corresponding linear measurements can firstly be demonstrated through practical measurement. Students may then be led to prove the relation for some particular cases.
			15	

UNIT 7

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
7	**Inequalities in algebra** *Objectives:* (1) To learn the simple laws of inequality. (2) To acquire the techniques of solving linear inequalities.	7.1 Simple inequality and its solution on the number line.	6	As an extension of the unit on "simple idea of ordering" some simple algebraic inequalities can now be considered. The point to emphasize here is the fact that for any two numbers a and b, one of the following statements must be true: $a=b$, $a>b$ or $a<b$ (the law of trichotomy). Before we consider the solution of a linear inequality, it is essential to demonstrate certain basic properties through examples. For $x \geq y$, we have $x+c \geq y+c$, $cx \geq cy$ $(c>0)$ and $cx \leq cy$ $(c<0)$. Students should realize that the law of transposing terms for equalities is also true for inequalities but when an inequality is multiplied by a negative number, the inequality sign should be reversed. The idea of open sentence in inequality can then be considered. The solutions of linear inequalities in one unknown such as $ax+b>0$ usually can be effectively represented on a number line with the use of coloured chalk. Teachers may also consider mentioning the terms "open interval" and "closed interval", though this is not necessary at this stage.
		7.2 Graphical solution of two linear inequalities in one variable.	6	For $b \leq a$, students should be able to combine $x \geq b$ and $x \leq a$ graphically and to write $b \leq x \leq a$. Care must be taken when the inequalities such as $x>a$ and $x<b$ are considered. Students should be able to state immediately that there is no value of x which can satisfy both inequalities simultaneously, rather than to graph the two inequalities and to think that the solution consists of two intervals.
			12	

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8	**Quadratic equations** *Objectives:* (1) To learn to factorize quadratic polynomials. (2) To learn to solve quadratic equations by factorization and by graphical method. (3) To learn to construct a quadratic equation when its roots are given as a reverse process.	8.1 Factorization of quadratic polynomials.	7	Methods of monomial factorization and grouping terms may be revised. Students are then given examples in using identities such as $x^2 \pm 2xy + y^2 = (x \pm y)^2$ and $x^2 - y^2 = (x+y)(x-y)$ for factorization purpose. For quadratic polynomials ax^2+bx+c where a, b and c are integers, such as $2x^2-5x-3$, the following way may be introduced. $2x^2-5x-3=(2x+1)(x-3)$ 2 ╲╱ +1 1 ╱╲ − 3 1 + (−6) = −5 It is advisable for teachers to first consider polynomials of the type x^2+bx+c, i.e. the special case in which $a=1$, and afterwards polynomials of the type ax^2+bx+c. After sufficient practice, students may be encouraged to write down the factors immediately by inspection.
		8.2 Solution by factor method.	5	As an introduction, teachers may find it profitable to explain the difference between a linear equation, such as $3x-12=0$, and a quadratic equation such as $x^2-3x+2=0$. It soon becomes evident that there are 2 values of x, i.e. 1 or 2 to make the open sentence $(x-2)(x-1)=0$ a true statement, and expressed in another way the equation $x^2-3x+2=0$ has 2 solutions, called the roots. After some worked examples, teachers may point out that for 2 numbers a and b, if $ab=0$ then either a or b must be 0. The technique used here is the factor method. While the coefficients in the quadratic equations must necessarily be simple to facilitate the factorization process, students should be make aware that not all quadratic equations can be solved this way and other techniques will be needed to treat equations like $x^2-3x+5=0$.

FORM III

UNIT 8

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
8				The sort of practical problems considered should be so chosen that they introduce the various pieces of manipulation which the class has to master. Care must be taken to ensure that the meaning of x be clearly stated before writing down an equation in x. Experience shows that when setting the class a problem exercise, it is usually profitable to spend some time discussing the questions beforehand, e.g. "What shall be taken for the unknown?" "What relation can be derived between the known quantities and the unknown?" etc. To illustrate the reverse process, teachers may draw a diagram as follows: Solving an equation to obtain roots ↓ $x^2-3x+2=0$ $(x-2)(x-1)=0$ $x=2$ or 1 ↑ Making an equation with given roots
		8.3 Solution by graphical method.	7	By drawing the graph of $y=ax^2+bx+c$ and reading the x-intercept(s), if any, it is possible to obtain approximate values of the roots of the quadratic equation $ax^2+bx+c=0$. Alternatively solutions may also be obtained by drawing the graphs of $y=x^2$ and $y=-\left(\frac{bx+c}{a}\right)$ Difficult cases when there are no real solutions can be shown vividly on the graph and readily understood by the students.
			19	

FORM III

UNIT 9

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
9	**Simple idea of probability** *Objectives:* (1) To understand the meaning of "chance" and to appreciate its use. (2) To learn the difference between theoretical and empirical probabilities. N.B.: Should emphasize the counting of the number of possible and favourable outcomes	9.1 Meaning of probability.	4	An alternative word for probability is "chance" and it is always represented by a number $p(0 \le p \le 1)$. This can be immediately understood by referring to realistic situations, for instance, the chance of throwing a six with a die, and so on. For combining probabilities, students can consider examples which are related to their daily experience. A student, for instance, knows that he has a 50-50 chance (i.e. the probability is $\frac{1}{2}$) of getting a ticket with an even number. Then a very natural question to ask is whether he knows the chance of getting another even numbered ticket next time. For abler students, such questions may take the form "What is the chance that the last 2 digits in the number on the ticket have even numbers?". After enough practice, classical examples on drawing balls out of a bag without replacing them and so on may be considered. The teacher may also discuss the chance of choosing 2 winners from 2 races, 3 races, and so on. However, it must be emphasized that if the idea of betting is brought up in class discussion, then the social significance and the ultimate outcome of losing one's money, rather than making a gain, should also be mentioned, the aim being to provide students with a certain amount of "equipment" as a means of protection.
		9.2 Experimental probability and theoretical probability.	4	Experiment here is to confirm the theoretical probability and students should note that most of the probabilities they come across in life such as the accident rate and the crime rate are empirical probabilities.
			8	

FORM III

UNIT 10

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
10	**Using trigonometry** *Objectives:* (1) To learn some applications of trigonometry. (2) To understand some simple methods of measuring inaccessible distances. (3) To learn the methods of locating a point on a plane. (4) To learn the techniques of resolving into right-angled triangles.	10.1 The use of gradients, angle of depression and angle of elevation.	4	Students should be led to understand that "gradient" is only a measure of the rising and falling of a straight line. This may be taught together with straight line equations. Many possible kinds of teaching aid can be constructed to measure the angle of depression and elevation, e.g. the clinometer. If possible some outdoor activities may be arranged to arouse interest. Throughout the unit we may come across many calculations. The use of calculators is desirable.
		10.2 Bearings on a plane.	4	The two different kinds of bearings such as 008° and N35°E are introduced. It may be much more interesting if each student is required to bring a compass to school so that problems involving bearings can be tackled in a practical manner.
		10.3 Two-dimensional problems soluble by analysis into right-angled triangles.	6	The success of this sub-unit depends mostly upon the teachers' analysis and the clarity of diagrams. Teachers usually find coloured chalk very helpful. A good knowledge in geometry as well as trigonometry should be the prerequisite. Hence frequent revision of geometric and trigonometric properties during problem solving is essential.
			14	

FORM III

UNIT 11

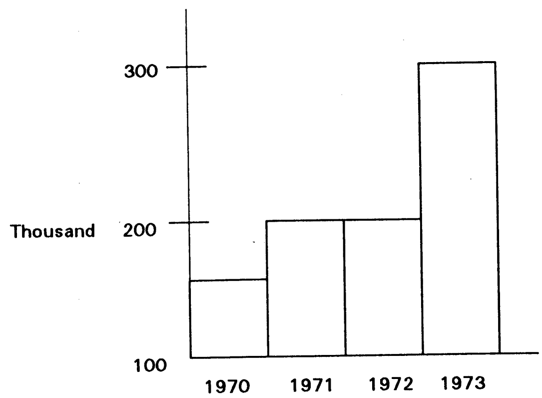
Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
11	**Measures of central tendency** *Objectives:* (1) To understand the meaning and significance of the central tendency of a distribution. (2) To learn some simple ways of measuring the central tendency. (3) To draw conclusions from these measures. (4) To apply these measures in daily life.	11.1 Mean, median and mode of discrete data.	8	The idea of average may be introduced by quoting several daily examples. This is one of the most commonly used measures of the "centre" of a distribution. Data from daily life should be given to students. They may be asked to locate the centre using whatever methods they like. Discussions on the different ways of finding the measure of the centre should be conducted before the formal definition of the arithmetic mean is given. The advantages of using the mean should also be emphasized. For an approximately symmetric set of data such as 13, 14, 15, 17, 18.5 the mean 15.5 gives the centre very effectively. Suppose that the salaries (in $) of a certain company are as follows: 1 000, 2 000, 3 000, 3 500, 4 500, 5 000, 30 000. The mean is 7 000 which is not a good measure of the centre. But the median is 3 500 which is found to be more satisfactory. Any extreme salaries do not cause the median to fluctuate much. The differences between the choices of the mean and the median should be clearly analysed. Examples to indicate the correct use of median should be provided for practice. Usually when a quick and approximate measure of the centre is needed, we take the mode. When we describe the style of dress or shoes worn by the "average woman", we mean the mode (most popular fashion). Daily examples should be provided for the demonstration of this idea. The understanding of the significance of the mean, median and mode is far more important than mathematical proof at this stage.
		11.2 Mean, median and modal class of grouped data.	5	It should be pointed out here that for a large set of data, it may be very difficult to calculate the mean. This leads to the idea of grouping the data first. The mean, thus obtained, is subject to the way of grouping and is only an approximation. This point should be emphasized.

UNIT 11

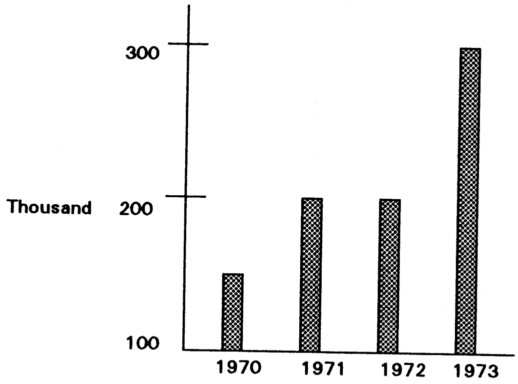
Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
11				The method of "assumed mean" is useful in reducing the burden of numerical calculation. For finding the median, it is also necessary to classify the data when the quantity is large. Why is a reasonable mid-value of a set of numbers, arranged in order of magnitude, obtained by dividing the histogram into two parts of equal areas? This is the underlying principle of making use of the histogram. It may be clarified by simple examples. No formal proof is required. Another graphical method of finding the median is to draw a cumulative frequency polygon. It should be very interesting to compare the results obtained by both methods for a distribution and also compare the underlying principles in both methods. The median for grouped data is determined by the use of cumulative frequency polygon/curve only. Colourful graphs may be very helpful in the teaching of these topics. For a large set of data, it is cumbersome to arrange it in order of magnitude. Usually we group the data in intervals. A discussion of the modal class, the interval with the highest frequency, may lead to a rough estimate of the mode.
			13	

UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12	**Uses and abuses of statistics** *Objectives:* (1) To understand how statistics is used in daily life. (2) To see the dangers of misinterpreting statistical data. (3) To understand the actual reasons why statistical data are so presented. N.B.: Daily life examples should be used to illustrate the uses and abuses of statistics. Project work will be useful.	12.1 Statistics in everyday life.	3	This sub-unit may be treated both as a general revision of statistics and as a vehicle for further examples. Teachers should prepare examples that are used in daily life as teaching aids. If time allows, students should be asked to collect or construct statistical graphs that serve a particular purpose.
		12.2 Misrepresentation of data.	5	Different ways of representing the same set of statistical data may give quite different impressions. Emphasis should be laid upon how some data are deliberately misrepresented to encourage a wrong conclusion. The following gives one example. A certain egg farmer presents his annual production report up to 1973.



Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12				Looking at the proportions of the areas, one may have the feeling that the egg production in 1973 is twice as much as in 1972 and four times as much as in 1970. In fact, it is not. We may however present the data by the following bar chart.



Intuitively, one may be led to conclude that there is only a slight progress in egg production. The teacher should give a complete analysis of the techniques used. Teachers will find that in this work overhead projectors are extremely useful.

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
		12.3 Telling lies with averages.	3	Throughout this unit, examples and discussions are the essential features. There are three different kinds of averages: the mean, the median and the mode. Each of these measures should be discussed. Attention should be paid to examples from daily life. The following gives one possible example on the "overworked" mean.

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12				A certain firm selling products A, B, C, D, E, at \$10, \$20, \$30, \$40 and \$100 respectively, wishes to raise the prices. Due to inflation, it is commonly accepted as reasonable to raise the selling price by 10%. The owner of the firm claims that the average selling price of his goods is still \$40 after inflation has increased his prices. This gives the impression that it is cheaper to buy in his firm. But the actual fact is as follows:

Before Inflation

A	B	C	D	E
\$10	\$20	\$30	\$40	\$100

$$\text{Average} = \frac{10+20+30+40+100}{5} = 40(\text{in } \$)$$

The firm increases the prices due to "inflation" (by far more than the acceptable 10%)

A	B	C	D	E
\$25	\$35	\$35	\$55	out of stock

$$\text{Average} = \frac{25+35+45+55}{4} = 40(\text{in } \$)$$

	Time Ratio
	11
Total:	160

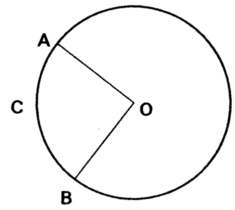
UNIT 1

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
1	**More about quadratic equations; surds** *Objectives:* (1) To acquire skills in solving quadratic equations by completing the square and by using formula. (2) To solve simultaneous equations, one linear and one quadratic. (restricted to algebraic method only) (3) To learn the relation between roots and coefficients. (4) To learn the rationalization process.	1.1 Completing the square.	3	By now students are quite familiar with the various techniques of solving equations including quadratic ones; using graphical or factor methods. Now they are introduced to a skill that requires thorough understanding of algebraic operations. Teachers should begin with examples like $x^2-8x+9=0$ and progress to examples like $3x^2-6x-14=0$, where the coefficient of x^2 is not unity. The steps in completing squares can be summed up on the board for ease of reference but students need not memorize the steps.
		1.2 Formula. Teachers may use the method of completing the square to derive the formula	5	Once the Students understand how should know the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for solving $ax^2+bx+c=0$ is derived, they should be able to reproduce it when needed. Teachers should ensure that students have no difficulty in applying the formula. One useful hint to see whether students know how to apply the formula is to ask them to write down the values of a, b and c first before they attempt to substitute them into the formula. When an equation such as $x^2+2x-1=0$ is considered, it is natural to leave the answers in surd form. When the students are quite familiar with the different techniques of solving quadratic equations, teachers may then ask if they could see any relations between the sum, product of the roots α, β and the coefficients of $ax^2+bx+c=0$. Then the relations $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$ should be introduced and proved. Exercises on calculating the values of expressions such as $\frac{1}{\alpha}+\frac{1}{\beta}$, $\alpha^2+\beta^2$, $\alpha^3+\beta^3$ and exercises in the formation of quadratic equations should be included.

UNIT 1

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
1		1.3 Simple problems using quadratic equations.	4	Problems requiring the solution of a quadratic are numerous in many school texts. Teachers should select those that have relation to students' experiences and preferably, have bearing on the practical application of mathematics. Examples can also be taken from physics or chemistry courses.
		1.4 Simultaneous equations: one linear and one quadratic.	4	It is desirable first to solve simultaneous equations in which one is linear and one is quadratic by using the graphical method. The graph of the quadratic should be plotted first and when a suitable straight line graph is added, the solutions may be readily obtained. Examples should be so chosen that one quadratic graph is used repeatedly to solve many quadratic equations. This will save students' time in plotting too many quadratic graphs. Teachers may find the graphical method useful in explaining why some quadratic equations have two roots, one root or no root at all. The algebraic method of substituting the linear equation into the quadratic equation should then be introduced and sufficient demonstration and practice should follow to ensure complete mastery of the technique.
		1.5 Rationalization of surds.	6	When an equation such as $x^2+3x-1=0$ is considered, it is natural to leave the answers in surd form. The term "surd" can then be explained and students are expected to be able to transform surd of any order into a surd of a different order. Addition, subtraction, multiplication and division of surds should be practised thoroughly before introducing the process of rationalizing the denominators of expressions of the form $\frac{1}{\sqrt{a}\pm\sqrt{b}}$.

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2	**Basic properties of a circle** *Objectives:* (1) To acquire an informal treatment of geometric argument. (2) To learn the basic properties of a circle, tangents to a circle, cyclic quadrilateral, and the tests for concyclic points. N.B.: In presentation of solution, brief reasoning is required. Restricted to nemerical problems only.	General approach.		In this unit, students will be expected to justify, follow and understand each deductive step of a proof, but no attempt should be made to build a formal and rigorously deductive structure based on carefully specified postulates and axioms. Students should not necessarily be expected to reproduce a formal proof of a geometric theorem.
		2.1 Chords and arcs of a circle.	5	The meaning of terms such as "arc", "segment", "sector" and "chor " should be reviewed. In order to differentiate between major and minor arcs, segments and sectors, it is simpler just to use an extra letter on the diagram.  e.g. Use $\overset{\frown}{ACB}$ rather than "minor $\overset{\frown}{AB}$" and sector OACB rather than "minor sector OAB" Teachers may emphasize that radii and chord form an isosceles triangle, and may use congruent triangles to show that the perpendicular to a chord from the centre of a circle bisects the chord. The fact that equal chords are equidistant from the centre follows.
		2.2 Angles in a circle.	10	In this sub-unit we are concerned with the angle at the centre, the angle at the circumference, angle in a semi-circle and angles in a cyclic quadrilateral.

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2				This work may be made a little more interesting if the central symmetry of a circle is used. An overhead projector is invaluable for this kind of demonstration. Alternatively use tracing paper, a pin and revolve the tracing paper to show that equal arcs subtend equal angles at the centre. This is also true for chords, but teachers may wish to validate the previous method by showing that "equal angles at the centre are subtended by equal chords". This may also be demonstrated using congruent triangles. However, it should be emphasized that, whereas arcs are proportional to the angles they subtend at the centre, chords are NOT. When demonstrating (the three cases) that the angle at the centre is twice the angle at the circumference and the angles in the same segment subtended by the same arc are equal, it is helpful for students to actually see that as the vertex of the subtended angle moves round the circle, the angle remains the same size. This may be done using a simple piece of apparatus such as a bead running on a wire arc and held by elastic bands. A piece of card can be used for the angle in order to show that it remains a constant size.
		2.3 Properties of cyclic quadrilateral and the tests for concyclic points.	11	The properties of cyclic quadrilateral such as (a) the opposite angles of a cyclic quadrilateral add up to 180°, and (b) if one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle should be proved. Sufficient exercises soluble by these properties should be given. The converses of the above properties constitute two tests for four concyclic points. It is also known that if the straight line joining two points subtends equal angles at two other points on the same side of it, then the four points are concyclic. These three tests for concyclic points should be discussed thoroughly and proved in detail.

UNIT 2

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
2		2.4 Tangent to a circle at a point and angles in the alternate segments.	11	The compasses construction forming the right angle should be done to emphasize the perpendicular property. However, in general, it is sufficient for students just to lay a ruler against the circle, at the point, in order to draw the tangent. Students are expected to know the basic properties of tangents. On completion of the teaching of angles in the alternate segments students should be exposed to an extensive array of miscellaneous exercises that make use of all the geometry done so far.
		2.5 A circle passing through three non-linear points.	3	The construction follows from the corollary to 2.1 para. 2, i.e. the centre of a circle lies on the perpendicular bisector of a chord. This construction not only emphasizes the theorem but is also another way of looking at the circumscribed circle of a triangle. The limiting case where the three points are collinear may be of interest to abler students.
			40	

UNIT 3

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
3	**Functions** *Objectives:* (1) To recognize the different kinds of numbers. (2) To understand the basic idea of a function. (3) To learn how to use the notation for a function. (4) To manipulate polynomials.	3.1 Number systems: integers, rational numbers, irrational numbers and real numbers.	5	This is essentially a revision of the elementary properties of integers and fractions and students are introduced to a new term: "rational numbers". An appropriate explanation of a rational number should be given according to the ability of the class. The characteristics of rational numbers when expressed in decimals should be demonstrated. e.g. Terminating decimal $\frac{2}{5} = 0.4$ $\frac{-3}{1} = -3.0$ Recurring decimal $\frac{1}{3} = 0.\dot{3}$ $\frac{2}{7} = 0.\dot{2}8571\dot{4}$ However, it should be noted that irrational numbers, e.g. $\sqrt{2}$, $-\sqrt{6}$, $\sqrt[3]{9}$, π, do not behave in this manner. The sets of rational and irrational numbers form the set of real numbers. Detailed and in-depth discussion of the real number system is NOT necessary.
		3.2 Concept of a function.	4	The idea of a function can be introduced as a relation between two varying quantities. Teachers may find that the idea of the number producing machine gives a useful pictorial representation in this context. However, students should see that functions transform numbers rather than generate them. Teachers should be sure that students do not try to solve functions as if they were equations. Teachers should give more examples of function such as sin x°, cos x° and log x, etc.

FORM IV and V

UNIT 3

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
3		3.3 Notation for a function: f(x) and y = f(x).	4	The notation f(x) should be introduced first and then a suitable letter, such as y, is introduced to denote f(x). In this way students can see that a function may be represented graphically on a coordinate plane. Once students are familiar with the notation, they may be asked to attempt questions like: Given $f(x) = x^2 + 2x - 1$, what are f(0), f(-2), and f(a-1)?
			13	

FORM IV and V

UNIT 4

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
4	**More about polynomials** *Objectives:* (1) To acquire skills in factorizing polynomials by factor theorem. (2) To find the H.C.F. and L.C.M. of polynomials. (3) To manipulate algebraic fractions.	4.1 Manipulation of polynomials.	4	At this stage, it is desirable to revise the manipulation of polynomials. Addition, substraction and multiplication of polynomials are standard work. Teachers may like to demonstrate that division of one polynomial by another does not generally lead to a polynomial. In preparation for further work, students should see and be able to recognize the general polynomial written in the form $a_0 + a_1x + a_2x^2 + ... + a_nx^n$.
		4.2 Remainder theorem and factor theorem.	4	Teachers are simply expected to illustrate the remainder theorem using use a quotient and divisor notation such as f(x) = (x-a)Q(x) + f(a). To consolidate the idea, students may also verify this theorem using the long division method. Then the factor theorem can be deduced.
	N.B.: Application of the theorem should be restricted to factorization of polynomials with known coefficients only	4.3 Factorization by factor theorem.	9	The use of the factor theorem becomes apparent when there is a need to factorize polynomials of degree three or higher. Students should also see that factorization will lead to the solution of the equation f(x) = 0. Functional notation should be used and the technique of using detached coefficients and synthetic division may be introduced. Questions of various types should be used to test thorough understanding and mastery of the factorization process. In using the factor theorem to factorize the polynomial $f(x) = a_0 + a_1x + ... + a_nx^n$, where a_0, a_1,..., a_n are integers, it is necessary to obtain a number α so that $f(\alpha) = 0$. A primitive method of getting α is by trial and error. When α is a rational number p/q, teachers should discuss the conditions for px-q to be a factor and deduce the relation between p, q, a_0 and a_n. In order to have a factor px-q, some rules have to be developed which serve as a better method of factorizing f(x).

UNIT 4

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
4				By factorizing x^3+1 and x^3-1, students may discover the identities $x^3\pm1\equiv(x\pm1)(x^2\mp x+1)$ and will accept the generalized result of $x^3\pm y^3\equiv(x\pm y)(x^2\mp xy+y^2)$ readily.
		4.4 H.C.F. and L.C.M.	3	The idea of finding H.C.F. and L.C.M. of integers may be revised and analysed. This immediately leads to the factorization method of finding H.C.F. and L.C.M. of polynomials. Emphasis should be laid upon the factorization method and other methods may be excluded.
		4.5 Manipulation of simple fractions.	5	Students are expected to master the technique of manipulating simple fractions using the four rules. Teachers may wish to find the L.C.M. of polynomials as a prerequisite to this topic; a direct manipulation of these fractions is also effective if done skilfully. It is advisable, therefore, to show a variety of examples that direct the students to the techniques of simplification rather than to the skill in manipulating long algebraic expressions.
			25	

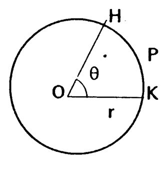
UNIT 5

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5	**Proportion and variation** *Objectives:* (1) To acquire further knowledge in rate, ratio and proportion. (2) To practise more in the use of rate, ratio, proportion and variation.	5.1 More on rate, ratio and proportion.	4	This is an extension of Unit 1 in Form II whence students were given the meaning of rate, ratio and proportion. However, students should make clear that rate provides a comparison of quantities not of the same kind and it bears a unit such as km per hour, while ratio compares quantities of the same kind and hence bears no unit. Students should be pointed out that ratio serves a better comparison between two quantities than using their difference. For example 10 is less than 20 by 10 just as 990 is less than 1 000 by the same amount. However, using ratio, one can have a better view. Since students have already seen some examples on rate, ratio and proportion, questions like in what ratio certain mixture of spirit could be mixed with water so as to decrease the percentage of spirit in the original mixture can be discussed. Students should make clear the idea of inverse ratio and hence its application. In problems on rate of working, the idea of treating the job as a unit quantity should be introduced for the manipulation.
		5.2 Algebraic manipulation of ratio and proportion.	5	Basic rules for ratio and proportion should be discussed and proved. For example if $\frac{a}{b}=\frac{c}{d}$, then ad = bc and so on. Afterwards, ideas can be extended to continued proportion, i.e. if $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots$, then each is equal to $\frac{ka+mc+ne+\ldots}{kb+md+nf+\ldots}$ where k, m, n, ... are constants. If it helps, numerical values could be used to show the equality. For example since $\frac{2}{4}=\frac{3}{6}$ one can see easily that $2\times6=3\times4$.

UNIT 5

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
5		5.3 Direct and inverse variation.	5	Students should make clear that variation refers to the change of certain quantity as some of its related quantities are changed. They should see that the change is regular and follows certain rule. Idea of dependence and independence can be shown by concrete examples such as the extension of spring with its acting loads. Examples like the payment of bus fares show the idea of direct variation and the sharing of a box of chocolates among some children shows the idea of inverse variation. The corresponding graphs of these two types of variation should be sketched and discussed. Special attention should be drawn upon the specific slopes of these graphs and hence a means to determine the variation constant.
		5.4 Joint and partial variation.	7	Examples in science like the related change in the volume, pressure and absolute temperature of an ideal gas show the idea of joint variation. On the other hand, the cost for making school badges with respect to the total number made illustrates the idea of partial variation. Many such examples in science and everyday life could be put forward to motivate students.
			21	

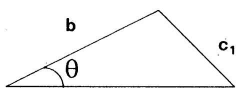
UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6	**More about trigonometry** *Objectives:* (1) To learn the concept of circular measure. (2) To learn the functions sine, cosine and tangent in the interval 0 to 2π, i.e. 0° to 360°, 0° to 90°. (3) To solve easy trigonometric equations. (4) To learn the area formula and the sine and cosine formulae of a triangle. (5) To learn the techniques of solving triangles.	6.1 Measurement of angles in radians.	2	Students should understand the meaning of a radian and the need of introducing it for use in further mathematics.
		6.2 Arc length and area of sector.	2	 Students have already learned the ratio method to find the arc length and area of sector. Now they should derive the formulae: $\overset{\frown}{HPK} = r\theta$ where θ is in radian measure. Area of sector HOKP $= \frac{1}{2}r^2\theta$ where θ is in radian measure.
		6.3 The functions sine, cosine, tangent and their graphs in the interval 0 to 2π, i.e. 0° to 360°, 0° to 90°	4	In defining the functions sine, cosine and tangent in the interval 0 to 2π, i.e. 0° to 360°, teachers may find it useful to use coordinates. Mnemonics and formulae may be used provided students can work out the trigonometric ratios of any angles or formulae from first principles. This is particularly important as students may use electronic calculators. In drawing graphs from 0 to 2π, i.e. 0° to 360°, 0° to 90° students may find it useful to choose the scale at intervals of $\frac{\pi}{6}$, i.e. 30°. Teachers can show students how and where the tangent graphs approach infinity.
		6.4 Easy trigonometric equations (solutions in the interval 0 to 2π, i.e. 0° to 360°), 0° to 90°	6	At this stage, the solution of trigonometric equations is best illustrated by examples. Initially some simple trigonometric functions may be presented graphically and the students led to discover the solutions of trigonometric equations from them. After some practice, students should be taught to use calculators to use tables to solve trigonometric equations, N.B.: Restricted to simple equations such as a sinθ =b, a cosθ =b, a tanθ =b. including quadratic equations which are factorizable. They are expected to be able to give all solutions in the interval 0 to 2π, i.e. 0° to 360°, 0° to 90°

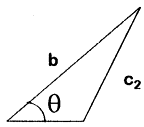
UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6				Although it is obvious that there is no limit to the number of possible solutions, the general solution of a trigonometric equation need not be considered at this level.
		6.5 Area of triangle as $\frac{1}{2}$bc sin A.	2	The formula is true for any two sides and the included acute angle. It can also be demonstrated that the formula is true for both acute and obtuse angles.
		6.6 The sine and cosine formulae of a triangle.	10	It is not difficult for students to see how the sine and cosine formulae are derived. Knowledge of the previous sub-unit can be used to derive the sine formula. When it comes to the ambiguous cases, that is, two sides and one non-included angle, teachers should explain with the help of separate diagrams such as Case (I) Case (II)

Case (I)



Case (II)



The cosine formula may be derived from the Pythagoras' Theorem or from the following three identities.

a = b cos C + c cos B
b = a cos C + c cos A
c = b cos A + a cos B

UNIT 6

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
6				It is worthwhile to note that the Pythagoras' Theorem is a special case of the cosine formula. It should be noted that the sine formula and the cosine formula together are sufficient to solve any triangle provided enough sides and angles are given to fix the triangle. Students should study elementary applications of these two formulae.
			26	

UNIT 7

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
7	**Arithmetic and geometric progressions** *Objectives:* (1) To recognize A.P. and G.P. (2) To learn the use of Σ notation. (3) To learn some properties of A.P. and G.P. (4) To learn the summation of A.P. and G.P.	7.1 Sequence and series.	2	Through the recognition of number patterns, students generally have no difficulty in understanding the meaning of sequence which simply means a string of numbers, sometimes with an easily recognizable pattern. However, students often find it difficult to define what a series is. It is suggested not to give a formal definition of series. More examples, especially of numerical type, should soon make the point clear. As a follow up to this topic, teachers may discuss some special number patterns such as triangular numbers, square numbers, rectangular numbers, etc. In-depth treatment of these patterns, however, should be avoided. The meaning of general term of a sequence should also be discussed and students are expected to know how to write down the first few terms of a sequence when the general term is given.
		7.2 A.P. and G.P.	4	Students should be able to recognize these two types of progressions and also be able to write down the general terms when the progressions are given. After enough practice, students may consider progressions in which the constant increment or multipier is negative, fractional or the square root of a certain number, etc. When a few terms of a progression are given, students should also know how to insert any number of terms between two given terms.
		7.3 Summation notation.	3	As a preparation for the study of summation problems of A.P. and G.P., teachers may find it useful to first introduce the notation $\sum_{i=1}^{n} x_i$. For the sake of abbreviating the notation further, it may be reduced to Σx_i and even to Σx. It can be seen that in actual manipulation, Σx is easier and more convenient to handle than others, provided no confusion arises.

UNIT 7

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
7				Properties such as (a) $\Sigma (ax \pm by) = a\Sigma x \pm b\Sigma y$ (b) $\Sigma (x \pm y)^2 = \Sigma x^2 \pm 2\Sigma xy + \Sigma y^2$ are useful in Unit 8. They may be introduced but proofs are not required at this stage.
		7.4 Summation of A.P. and G.P.	5	The summation formula may be derived using the Σ notation or other method but students are not expected to reproduce the proofs. Practical examples should then be considered. The case of infinite G.P. may be briefly discussed and illustrated by examples. Thorough treatment of infinite series is not expected.
			14	

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8	**Probability and statistics** *Objectives:* (1) To learn more about probability and statistics. (2) To apply the basic laws in probability to simple problems. (3) To learn the concept and measures of dispersion of a distribution. (4) To learn some applications of the mean and the standard deviation.	8.1 Addition and multiplication laws.	7	This sub-unit is a continuation of Unit 9 in Form III. After students are acquainted with the idea of probability, it is natural for them to consider more complicated problems. Examples like tossing coins, throwing dice and drawing cards illustrate the concepts of "mutually exclusive" and "independent events". It would be harmful to try to give a formal definition at this stage. The addition law may be illustrate by numerical examples. The concept of the multiplication law may be interpreted as a fraction of a probability. Initially, it is desirable for students to tackle problems from the common sense point of view. Finally teachers should summarize the techniques in the form of the above two laws. For demonstration, numerical examples should not only involve theoretical probabilities but also experimental probabilities. For example: The probability of having a boy is 0.55. What is the probability of having a boy and a girl?
		8.2 Weighted averages.	6	The mean of a frequency distribution of Unit 11 in Form III is in fact a mean weighted by its frequencies. It may be regarded as a weighted average (mean). The concept of weighted average is commonly known in practical life. Teachers should find no difficulty in collecting examples in daily life to illustrate the application of weighted average. *Example:* The following is the examination report of two students T_1 and T_2,

	T_1	T_2	No. of periods per week
Chinese	70	90	10
English	65	60	8
Maths	85	61	5

UNIT 8

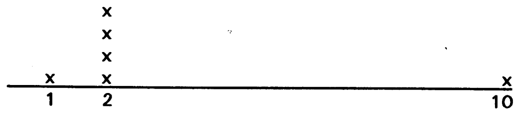
Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8				Which student did better in the examination? In order to answer such a question we consider their average scores. The average score of $T_1 = \frac{70+65+85}{3} = 73.33$ The average score of $T_2 = \frac{90+60+61}{3} = 70.33$ We say that T_1 did better than T_2. On the other hand, if we take the weighted averages with the number of periods per week as weights then the weighted average of $T_1 = \frac{70\times10+65\times8+85\times5}{10+8+5} = 71.52$ the weighted average of $T_2 = \frac{90\times10+60\times8+61\times5}{10+8+5} = 73.26$ We then say that T_2 did better than T_1. Can we have other conclusions?
		8.3 Measures of dispersion: range, mean deviation, variance and standard deviation (grouped and ungrouped data).	6	In statistics, we accept that a set of data cannot be perfectly uniform. Teachers may use the following examples for illustration: (a) the weights of children having the same height; (b) the measurements of the diameter of a circle by different children; (c) the heights of the ceiling as estimated by the students in the class; (d) the monthly expenditure of families of the same size; (e) the prices of the same brand of rice in different shops, (Students may be asked to collect the data themselves so that they may have a better feeling of the variability.)

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8				

To measure the variability (dispersion) of a set of data, we may use the range, the mean deviation, the variance or the standard deviation.

The simplest measure of dispersion is the range, which is defined as the difference between the largest and the smallest values in a set of data. The disadvantage of this measure is that it does not take the intermediate values into account. Thus, the following distributions



have the same range, but certainly not the same dispersion.

A better measure is the mean deviation $= \frac{1}{n}\Sigma f|x - \bar{x}|$.

The mean deviation directly gives the average difference of each number from the mean. Teachers should give a full explanation (including the absolute sign) as to how the mean deviation can measure the dispersion of the distribution. However, as the absolute sign is very difficult to handle in mathematical computation, we consider the

variance $= \frac{1}{n}\Sigma f(x - \bar{x})^2$.

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8				

This is also regarded as a measure of dispersion. The idea of squaring the difference from the mean is to eliminate the absolute sign. However, the variance has a disadvantage of having a higher dimension.

To reduce it to the same dimension as the data, it is quite natural to use the

standard deviation(s) $= \sqrt{\frac{1}{n}\Sigma f(x - \bar{x})^2}$.

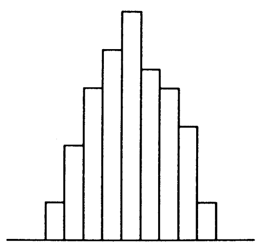
A better measure is the standard devation

$$\sqrt{\frac{1}{n}\left[f_1(x_1-\bar{x})^2 + f_2(x_2-\bar{x})^2 + \ldots + f_n(x_n-\bar{x})^2\right]}$$

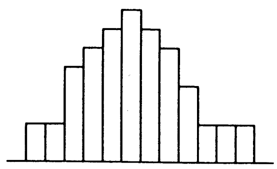
The relation between the graph of the distribution and its standard deviation should be shown.

For example:

Small value of s



Large value of s



At this stage, we shall only consider the "spread" of the graphs with reference to the standard deviation.

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching

8

Students should acquire "feeling" for the meaning of mean and standard deviation. This is much more important than expertise in their calculation.

Each time they consider a distribution, they may be asked "What percentage of the readings lies within one standard deviation of the mean, within two standard deviations,...?". We may mention that for many of the distributions we meet in daily life, business and industry, especially those with a bell shape, about $\frac{2}{3}$ of the data lies within one standard deviation from the mean, and almost all within three standard deviations. However, a treatment on normal distribution should not be included.

8.4 Method of computing standard deviation (grouped and ungrouped data).

Time Ratio: 9

N.B.: Students may use calculators to compute standard deviation

With certain calculators, we can obtain the value of s by simply pressing the s-key (or σ-key). However, students should also know how the value of s is computed. Obviously, s may be computed directly from its definition. An alternative method is to use the following formula:

$$s = \sqrt{\frac{1}{n}\sum fx^2 - \left(\frac{1}{n}\sum fx\right)^2}.$$

Students are not expected to know how to derive the second formula from the first, but a discussion of the derivation led by teacher will give them faith that the formulae are equivalent.

(Teachers should note that the formula used in many calculators for s is

$s = \sqrt{\frac{1}{n-1}\sum f(x-\bar{x})^2}$, since it is a better estimate of the standard deviation of the population from which a sample has been taken. Some calculators provide separate keys for calculating s using these two different formulae.)

UNIT 8

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching

8

8.5 Application of standard deviation.

Time Ratio: 9

In the teaching of the standard deviation of a distribution, greater emphasis should be placed on the understanding of standard deviation as a measure of dispersion (variability) rather than on the numerical calculation from a complex set of data.

Extensive quantitative applications of the standard deviation are not expected but the following examples may be taken as illustrations:

Example 1 (Standard scores)

The standard score $z = \frac{x-\bar{x}}{s}$ is a conversion of raw scores for comparison purposes. Teachers should explain the difference $x-\bar{x}$ and the ratio $\frac{x-\bar{x}}{s}$. The standard score is commonly used in examinations for comparison of students' abilities in different tests. Let us consider the marks in History and Geography of a class of ten students. If a certain student D scores 82 in History and 69 in Geography, in which subject does he do better?

Student	The scores History	Geography
A	95	60
B	90	50
C	80	55
D	82	69
E	79	61
F	60	68
G	70	70
H	85	59
I	75	71
J	68	72

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8				

Apparently the student has done relatively better in History. But, if we investigate the scores of the class carefully, we may have a completely different picture.

$\bar{x}_1 = 78.4$ $\qquad$ $\bar{x}_2 = 63.5$

$s_1 = 9.99$ $\qquad$ $s_2 = 7.17$

$$z_1 = \frac{82 - 78.4}{9.99} = 0.36 \qquad z_2 = \frac{69 - 63.5}{7.17} = 0.77$$

It is natural to assume that the performance of the class is consistent in the two tests. We can then quite reasonably say, from another point of view, that the student does better in Geography than in History.

Example 2 (Life time of electric bulbs)

As a result of tests on electric light bulbs, it was found that the lifetime of a particular make was distributed symmetrically about the mean. The mean lifetime was 2 000 hours and the standard deviation was 80 hours. What proportion of bulbs can be expected to have a lifetime

(a) of more than 1 920 hours, and
(b) of more than 2 080 hours?

Example 3 (Standard deviation as an indication of precision)

Two instruments, A and B, are used to measure a quantity for the same number of times (20 times with each instrument, say). A gives a standard deviation of 2.6 units while B gives a standard deviation of 1.6 units. Which instrument is more precise?

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
8			37	

Example 4 (Use of standard deviation to measure non-uniformity)

Each of two factories employs the same number of workers. When the monthly salaries of the workers are processed, it is found that the standard deviation of the salaries for the workers of Factory A is $200 while that of Factory B is $50. Which factory offers more uniform salaries to its workers?

Example 5 (Use of standard deviation for setting up acceptable limits)

Bags of sugar are filled to the nominal weight of μ kg by a machine. The actual weights of the sugar in the bags are thus not necessarily equal to μ kg, but can be somewhat higher or somewhat lower. If a bag weighs much below its nominal value, the customer may claim refund. Usually, the limit for underweight is expressed as the nominal weight minus a certain multiple of the standard deviation (e.g. μ-3σ). Thus, if the nominal weight is 1 kg and the standard deviation is 20g, the customer may claim refund for a bag of sugar less than 0.94kg.

UNIT 9

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
9	**Inequalities** *Objectives:* (1) To learn the graphical representation of algebraic inequalities on number lines and on a plane. (2) To learn the solution of algebraic inequalities and to apply it to linear programming problems. (3) To learn the graphical or tabulation method to solve quadratic inequalities.	9.1 The number line and the solution of linear inequalities in one variable.	4	This is a revision of the material taught in F.I to F.III. Students should begin with examples like x> or x<b and then proceed to consider the general inequality ax+b>c. Experience shows that it is effective to mark the intervals on the number line by coloured chalk. Inequalities involving "≥" should also be discussed in a similar way. The terms closed and open intervals should be introduced and discussed. There are many ways to mark the closed and open intervals on a number line. The example below indicates one way.

$-4<x<3$

open interval

$-4 \le x \le 3$

close interval

$-5<x \le 4$

half open (or half closed) interval

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
		9.2 Quadratic inequalities in one variable.	4	Graphical representation is useful in helping may help students to understand the solutions. A table such as the one below leads easily can also lead to the solution of (x+5)(x−7)<0 or (x+5)(x−7) ≥ 0.

x	...	-5	...	7	...
(x+5)	-	0	+	+	+
(x-7)	-	-	-	0	+
(x+5)(x-7)	+	0	-	0	+

UNIT 9

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
9				For able students, they can also consider examples like: $(x-1)(x+2)(x+4)(x-7)<0$, or $\frac{(x-1)(x+2)}{(x+4)(x-7)} \ge 0$ If they do, the expression on the left should be given in factorized form.
		9.3 Solution of linear inequalities in two variables.	7	It is recommended to introduce the sub-unit from easy inequalities of one variable to more complicated ones in two variables. The following is a possible sequence of examples to be considered: (a) y>c (b) y>d (c) x>c and y>d (d) x>c or y>d (e) x+y>a (f) ax+by>c Inequalities involving "≥" should also be discussed in a similar way. Graphical representation of solution in each case should be presented neatly on the board and marked with coloured chalk if available. The use of overhead projector, graph board, magnetic graph board or pin board are desirable. In cases where two or three linear inequalities (normally not more than three) are considered, teachers may find it convenient to use the terms "union" and "intersection" informally.
		9.4 Application to linear programming.	4	This demonstrates the use of linear inequalities. School texts are full of graphical examples in linear programming involving two variables. Teachers should try to introduce problems that are more practical and meaningful. A theoretical approach to linear programming is not necessary and discussion should be restricted to graphical treatment.

UNIT 10

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
10	**Application of trigonometry** *Objective:* To apply trigonometric knowledge in solving two and three dimensional problems. N.B.: For 10.2, examples should be restricted to very simple ones	10.1 True bearings.	2	In Form III, students were introduced to the two principal methods of indicating the direction: Compass bearings and True bearings. For calculation at this level, students are expected to use true bearings. Simple problems involving bearings of one point from another or vice versa should be discussed.
		10.2 Easy problems in two and three dimensions.	9	There are many practical problems which involve sine and cosine formulae, both in two and in three dimensions. In particular, problems involving the line of greatest slope would be of interest to students. For three dimensional problems, students should investigate how to solve simple problems involving (a) the angle between two intersecting lines, (b) the angle between a line and a plane, and (c) the angle between two intersecting planes. Only those problems reducible to right-angled triangles are to be considered. Teachers may also find wire-models or 3-D teaching aids useful for explanation and illustration.
			11	

UNIT 11

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
11	**Coordinate treatment of straight lines and circles** *Objectives:* (1) To learn the idea of loci as a basis for further work on simple conics. (2) To discover the relationship of the slopes of parallel lines and that of perpendicular lines.	11.1 Establishing the concept of locus.	2	Approach this idea in as many practical ways as possible e.g. paths of a moving point, a moving line, a moving area and moving objects.
		11.2 Construction of loci within a plane.	5	Construction of the locus of a point moving equidistantly from (a) a fixed point, (b) two fixed points, (c) a fixed line, and (d) two fixed lines. Using simple apparatus such as string, spirograph and mecanograph, students may construct parabola, ellipse, cycloid and a variety of other loci. The important thing is to select apparatus where the scribe moves according to the given conditions.
	(3) To look at the circle from the coordinate point of view, and to study the equation of circle. (4) To understand the possible intersections between a straight line and a circle.	11.3 Straight line, gradient, parallel and perpendicular lines.	8	Revision of $y=mx+c$ emphasizing that the gradient (slope) is also the tangent of the angle θ made with the x-axis. Now that $\tan\theta$ has been defined for the general angle, it is easily demonstrated that $m=\tan\theta$ for θ obtuse as well as acute. Hence, lines are parallel when $m_1=m_2$ for θ obtuse as well as acute. To demonstrate that for perpendicular lines $m_1m_2=-1$ use the theorem about exterior angle of a triangle and $\tan(90°+\theta)=\frac{-1}{\tan\theta}$. Multiple angles should not be used at this stage. The case $\theta=90°$ may be discussed separately. Students should be encouraged to discover that parallel lines have the same slope and the product of slope of perpendicular lines is -1, but the proof is not required. Applications to find equations of straight lines parallel/perpendicular to a given line can also be mentioned. This work used with the mid-point of a line segment opens up further links with other geometry units. Exercises relating to properties of plane figures, such as the diagonals of a parallelogram bisect each other, should give students an awareness of the usefulness of the coordinate system.

FORM IV and V

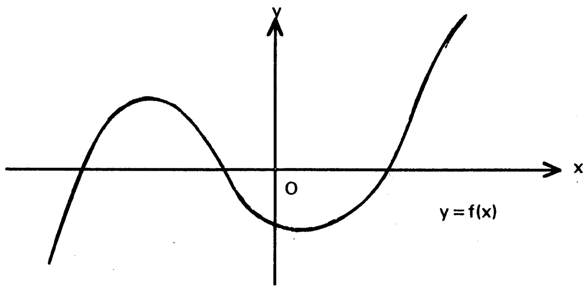
UNIT 11

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
11		11.4 Equation of a circle with centre at the origin.		Students should be led to derive the equation of a circle with centre at the origin and radius r from the distance formula. After several examples, they will discover the equations take the form $x^2+y^2=r^2$. Given an equation as in the above form, students should recognize that it is a circle with centre at the origin and radius equals to r.
		11.5 Equation of a circle in general position.	6	Teachers may then consider a more general situation, viz., the centre is not at the origin but with coordinates (h,k). After some practice, students will discover the equations can take one of the following two forms: $x^2+y^2+Dx+Ey+F=0$, or $(x-h)^2+(y-k)^2=r^2$. Given an equation as in either of the above forms, students should recognize that it is a circle. They should also know where the centre lies and the length of its radius. The equation of a circle passing through three non-linear points should also be discussed.
		11.6 Intersection of a straight line and a circle.	6	Does a straight line usually cut a circle at two points? Teachers should discuss all the possible cases in relation to the roots of a quadratic equation, especially when the quadratic equation has a double root. The idea of tangency in geometry may be given in terms of algebraic condition $b^2-4ac=0$. Further exercises on finding the equations of tangents under simple conditions may be given for abler students.
			30	

FORM IV and V

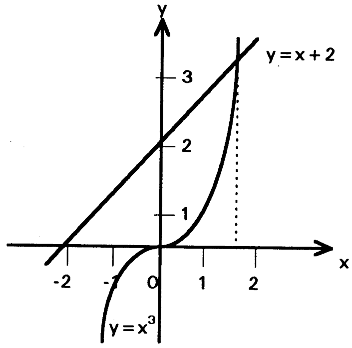
UNIT 12

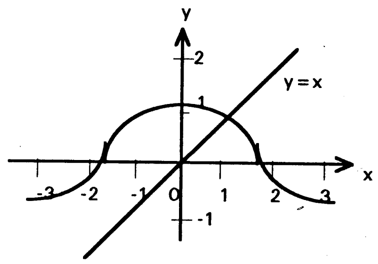
Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
12	**Approximate solution of simple equations** *Objectives:* (1) To revise and extend the idea of representing equations by graphs. (2) To learn how to solve simple equations by graphical methods. (3) To learn the method of bisection for solving simple equations to a prescribed degree of accuracy.	12.1 Graphical solution of equations.	5	By now, students should be able to solve quadratic equations (with real roots). Taking them a step further, teachers may lead students to consider the solution of other simple equations such as x = cosx, $x^3-x-2=0$. Many of these equations cannot be solved algebraically to give exact solutions, but most of them can be solved graphically to give approximate solutions. Teachers should review the graphical representation and solution of quadratic equations studied in Form III (Sub-unit 8.3). Several graphical methods are available for solving simple equations. One method is to arrange the equation in the form f(x) = 0. With the help of calculators, it is relatively easy to make a table of values of f(x) for suitable values of x and plot the graph y = f(x). At a real root of the equation f(x) = 0, y = 0 and hence the root is the value of x where the graph crosses the x-axis and this can be read from the graph.



UNIT 12

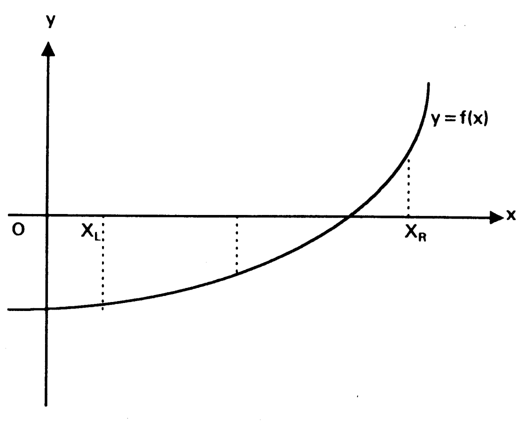
Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12				Another commonly used method is to arrange the equation in the form x=g(x) and plot the graphs y=x and y=g(x). The points of intersection of these graphs then give the roots of the equation. More generally, we may arrange the equation f(x)=0 in the form g(x)=h(x) and the roots are given by the points of intersection of the curves y=g(x) and y=h(x). Teachers should give a comparison of the methods. Note that answers can be read more easily and more accurately if the curves intersect almost at right angles, and this may serve as one criterion for choosing the graphical method used.





UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12		12.2 Method of bisection.	10	While graphical methods work well for many simple equations, they have the disadvantage that the accuracy of the answers cannot be controlled easily. A simple method which can be used to improve the accuracy of the graphical solution and to give the solution to a prescribed degree of accuracy is the method of bisection. In this method, we first find an interval which "brackets" the root and then reduce the "bracketing" interval successively by half until finally the root is "trapped" within an arbitrarily small interval.



UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12				

For a simple root, a bracketing interval $x_L < x < x_R$ has the property that $f(x_L)$ and $f(x_R)$ have opposite signs, i.e. $f(x_L)f(x_R) < 0$.

The teacher may introduce the method by the following example:
Find the real root of

$x \log x - 1.2 = 0$

correct to two decimal places. (The log is to base 10.)

A graphical method may be employed to find the first approximation of the root. Alternatively, the following table may serve to find the first bracketing interval:

x	f(x) = x log x -1.2
1	-1.2
2	-0.598
3	0.231

∴ the true root x_0 must lie between 2 and 3 and hence we calculate f(2.5) next.

UNIT 12

Unit No.	Basic Content/Objectives	Detailed Content	Time Ratio	Notes on Teaching
12				

The table below shows the working and should be easy to follow:

x	f(x) = x log x−1.2	Observation and further step
2.5 3.0	−0.205 1 0.231 4	∴ $2.5 < x_0 < 3.0$ Next find f(2.75);f(2.75) = 0.008 2
2.5 2.75	−0.205 1 0.008 2	∴ $2.5 < x_0 < 2.75$ f(2.625) = −0.099 8
2.625 2.750	−0.099 8 0.008 2	∴ $2.625 < x_0 < 2.750$ f(2.688) = −0.045 7
2.688 2.750	−0.045 7 0.008 2	∴ $2.688 < x_0 < 2.750$ f(s.719) = −0.018 8
2.719 2.750	−0.018 8 0.008 2	∴ $2.719 < x_0 < 2.750$ f(2.735) = −0.004 9
2.735 2.750	−0.004 9 0.008 2	∴ $2.735 < x_0 < 2.750$ f(2.742) = −0.001 2
2.735 2.742	−0.004 9 0.001 2	

Since $2.735 < x_0 < 2.742$, $x_0 = 2.74$ correct to 2 decimal places.

A sequence of sketches accompanying the steps will illustrate the process still better.

UNIT 12

Unit No.	*Basic Content/Objectives*	*Detailed Content*	*Time Ratio*	*Notes on Teaching*
12				*Notes* (a) At this level, only equations with simple roots will be considered. Equations with equal roots, and more generally, cases where the bisection method does not work, may be discussed qualitatively. Students are not expected to handle such cases themselves. (b) A detailed discussion of the advantages and disadvantages of the bisection method may not be fully appreciated by students at this level, as no other numerical methods have been introduced for comparison. However, after working through several examples, students may realize (i) that the method should work for most of the simple equations, and (ii) a considerable number of iterations may be required to achieve a specified degree of accuracy.
			15	
		Total:	273	